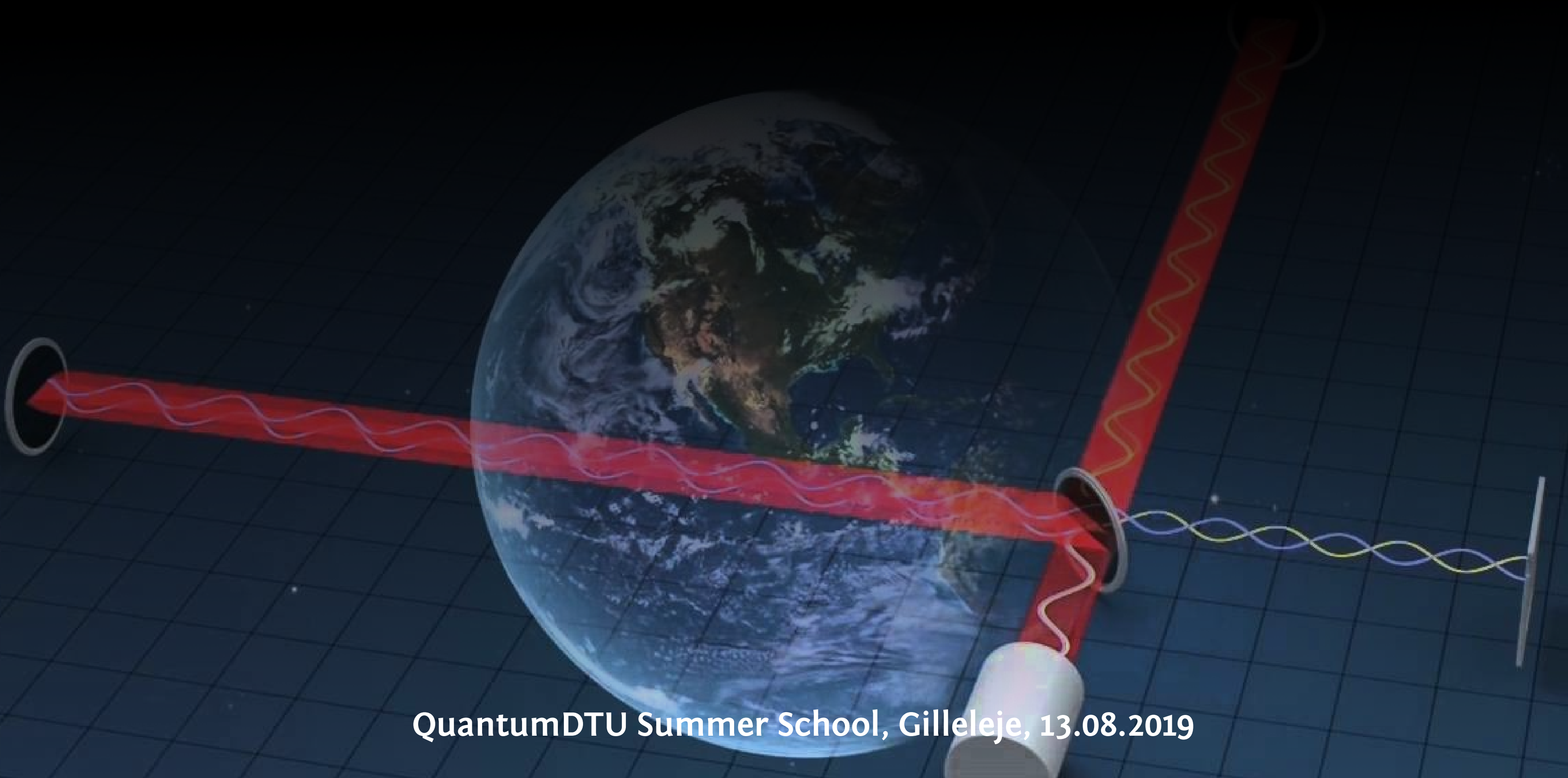
Noisy Quantum Metrology

Jonatan Bohr Brask
DTU Physics



QuantumDTU Summer School, Gilleleje, 13.08.2019

Why precision measurements?

- Triggers science breakthroughs.

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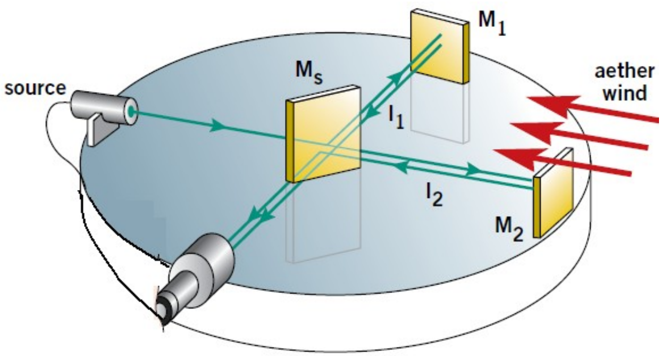
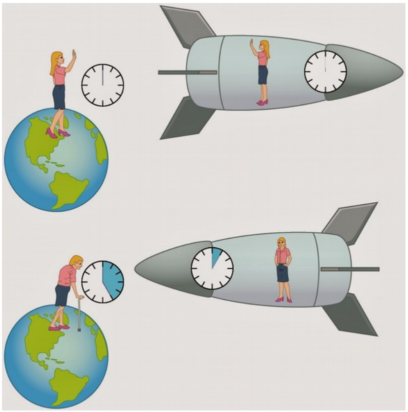


Michelson-Morley experiment, 1887



Special relativity

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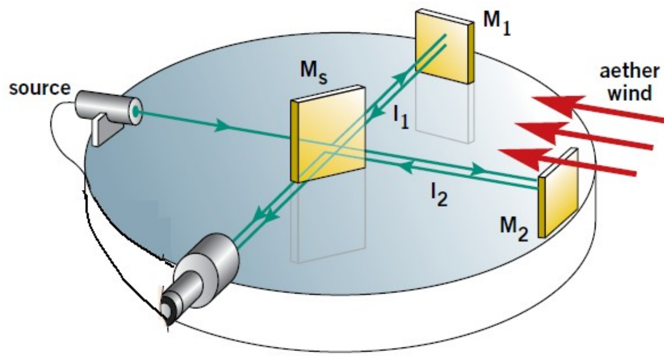

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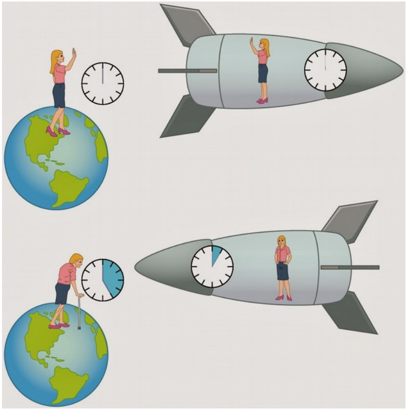
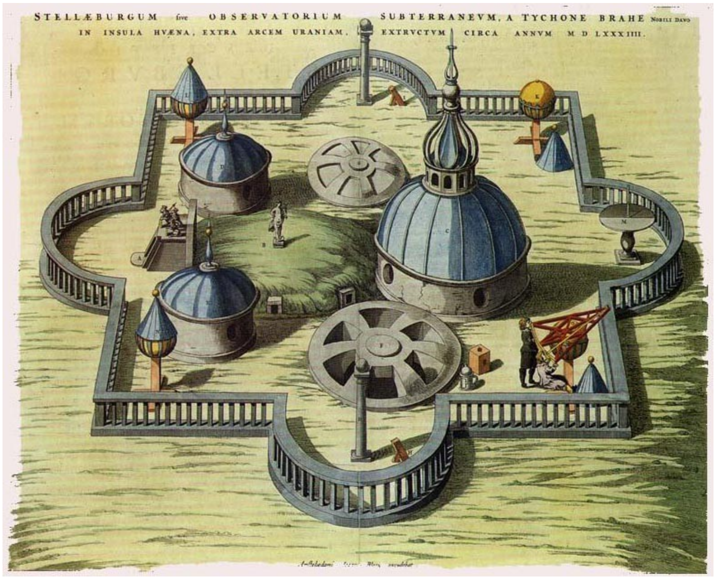
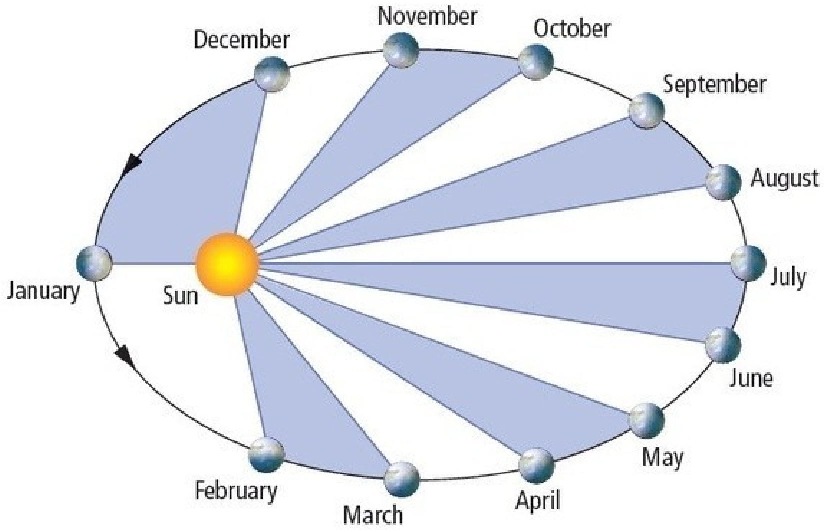


Special relativity



Tycho Brahe astronomical observations, late 1500s



Kepler’s laws, changing heavens

Why precision measurements?

- **Enables technology**

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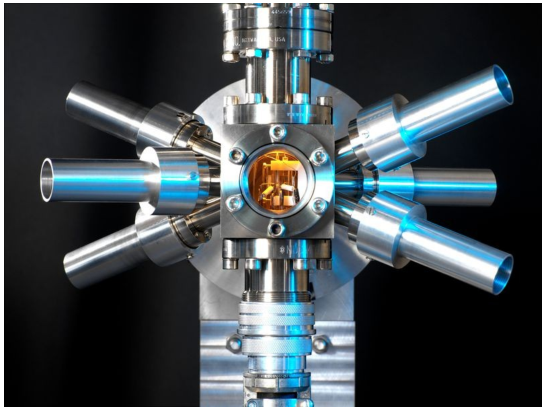

Precise time keeping (atomic clocks)

GPS

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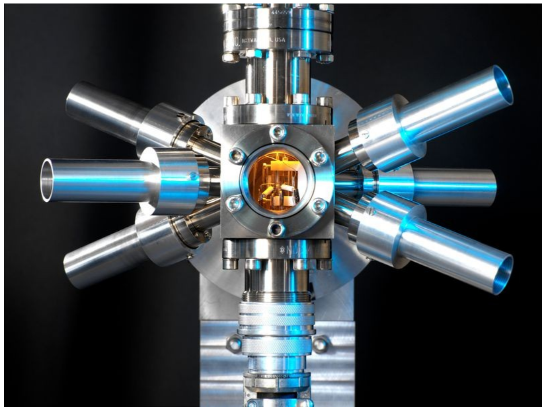

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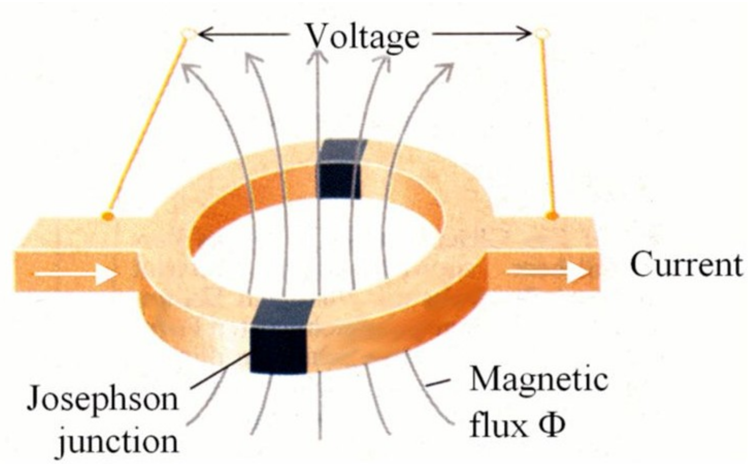

Precise magnetometry (SQUIDs)

Magnetoencephalography

Why quantum?

- **Better precision without increasing probe size / measurement time.**

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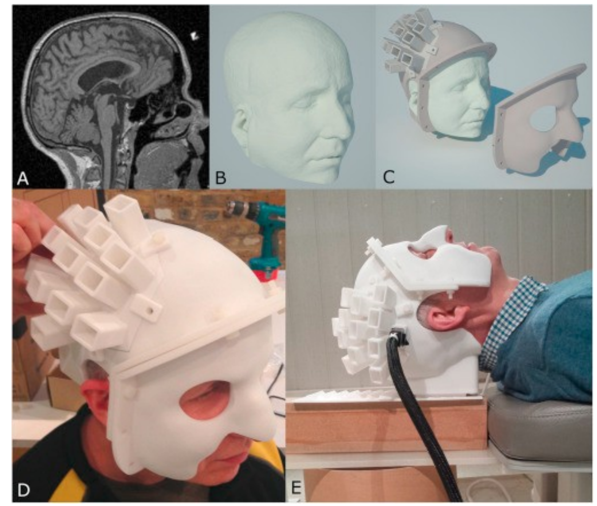
Atomic vapor magnetometry / Nitrogen-vacancy centers in diamond



Boto *et al.*, NeuroImage, 149, 404 (2017)

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- **Reduce sensor size w. quantum effects.**

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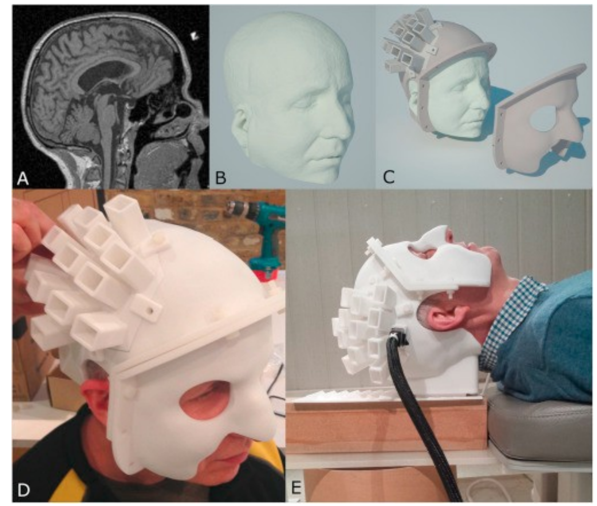

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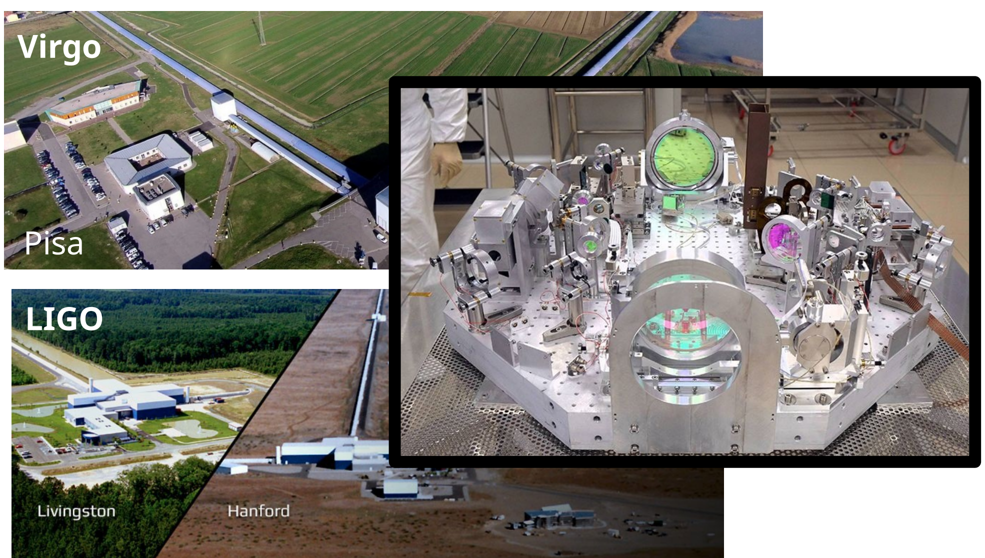
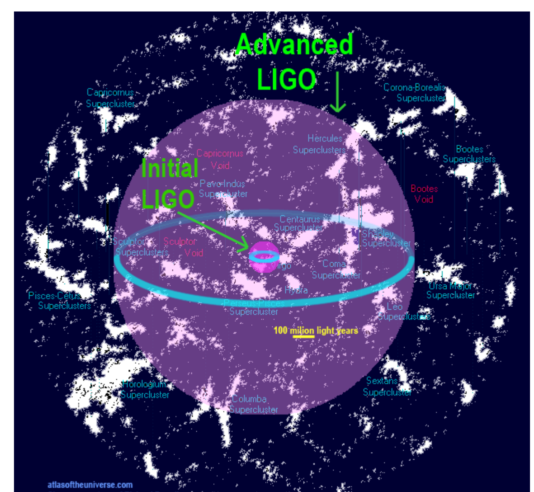


Squeezed light



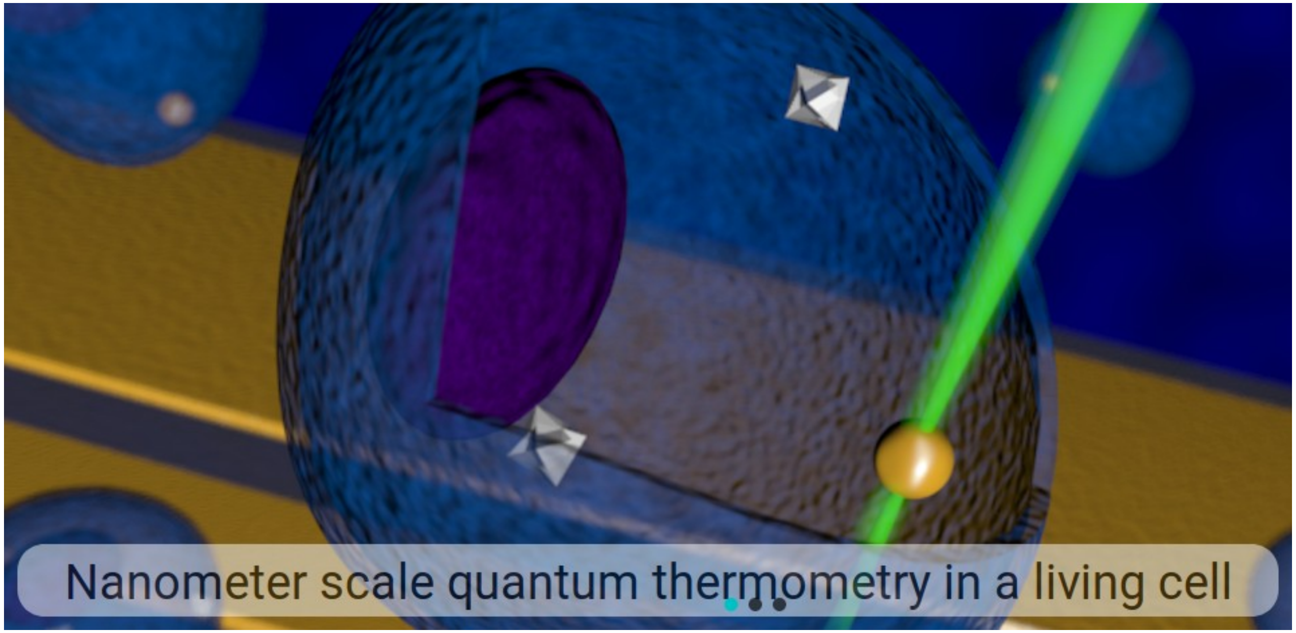
- **Better sensitivity w/o increasing light power.**

Why noisy?

Because any real system is...

Need to demonstrate that quantum techniques give an advantage under realistic conditions.

We would like to apply quantum metrology not just for well controlled lab physics but also for applications in more noisy environments.



Kucsko *et al.*, Nature 500, 54 (2013).

Outline

Noiseless quantum metrology

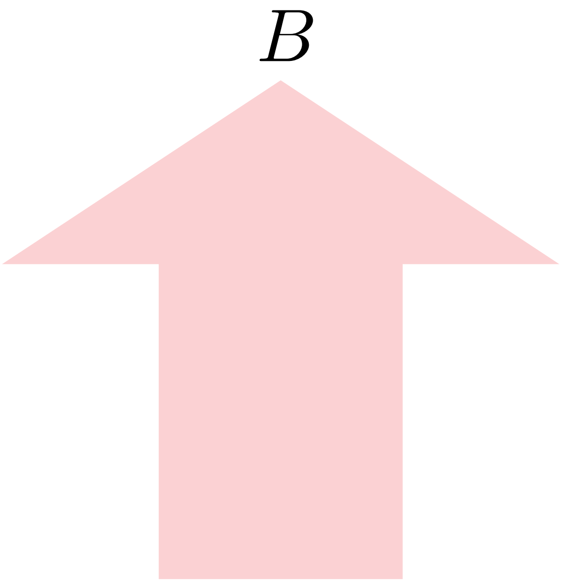
- Parameter estimation
- Precision scaling
- Fisher information
- Standard quantum limit
- Heisenberg limit

Noisy quantum metrology

- No-go results
- Directional noise
- Quantum error correction

Parameter estimation - examples

Magnetometry



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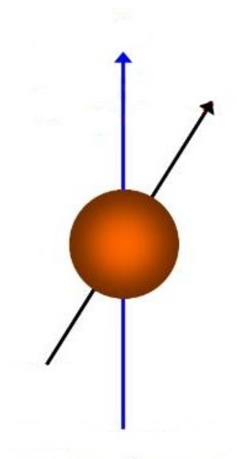

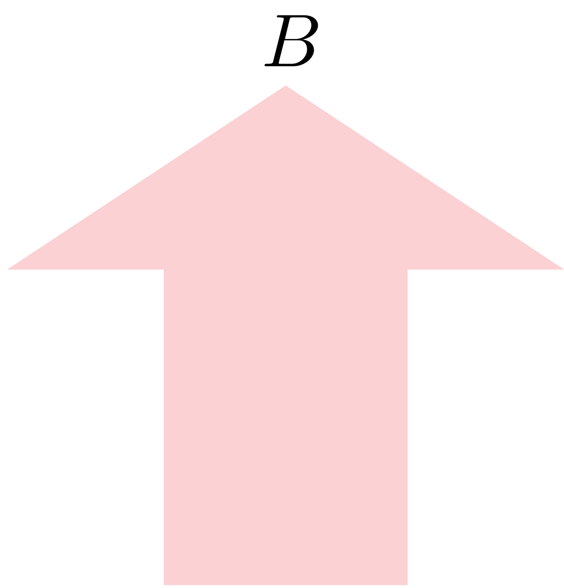

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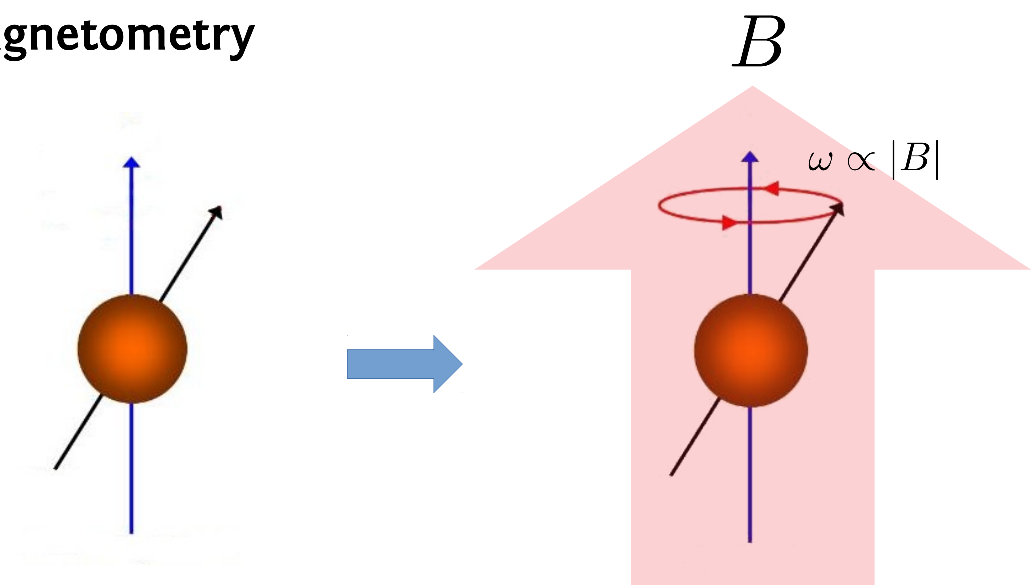

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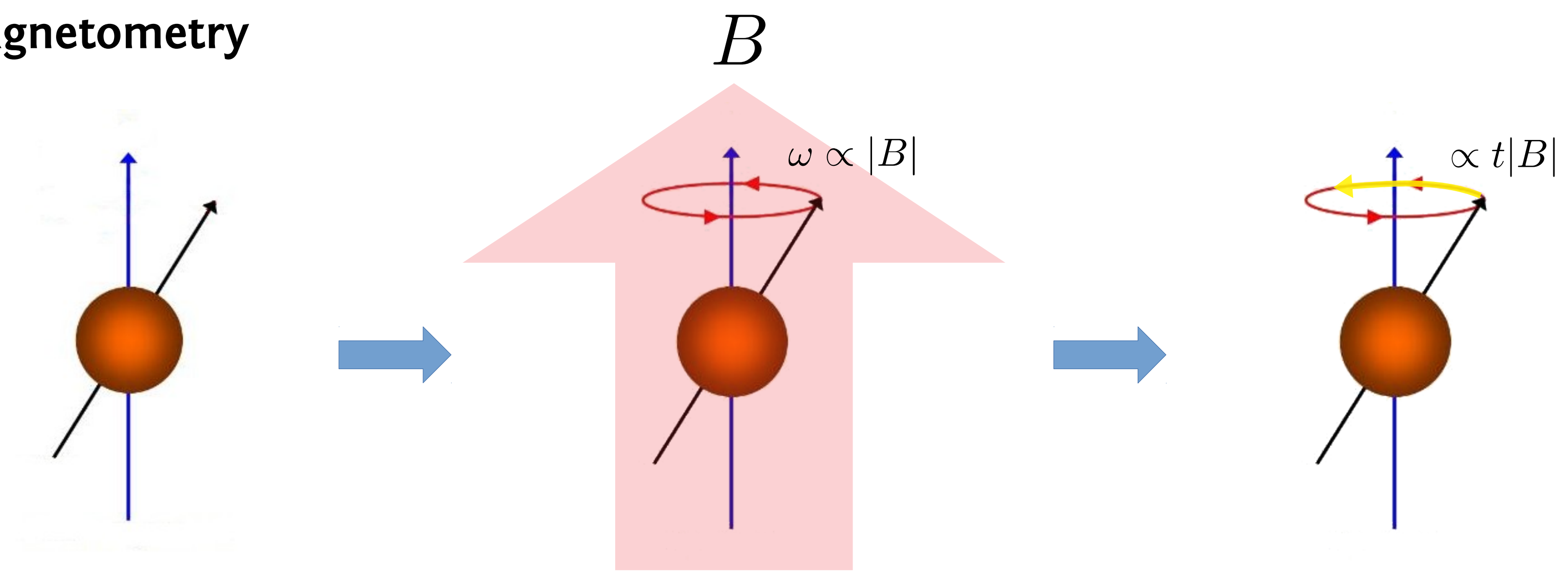

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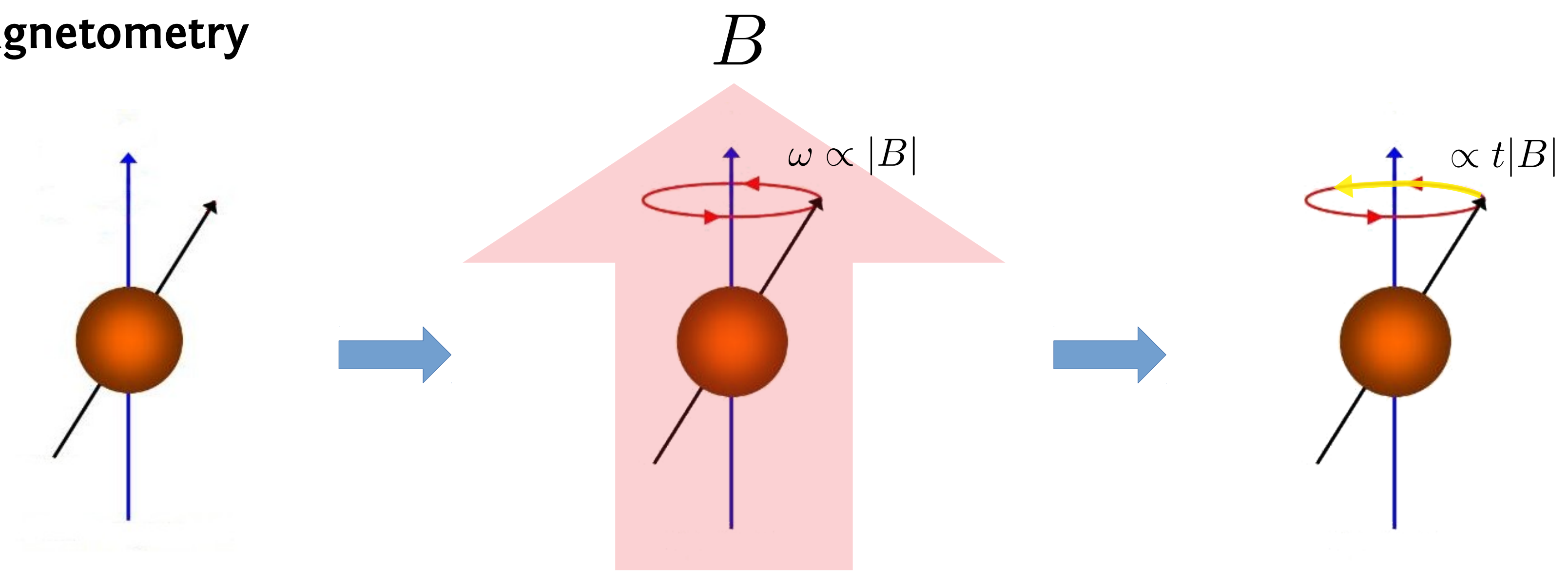
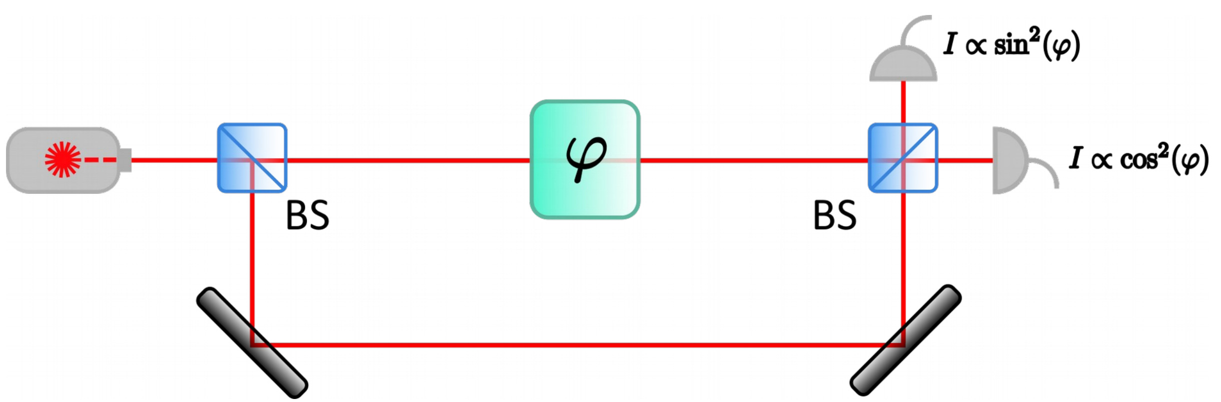


Optical phase estimation



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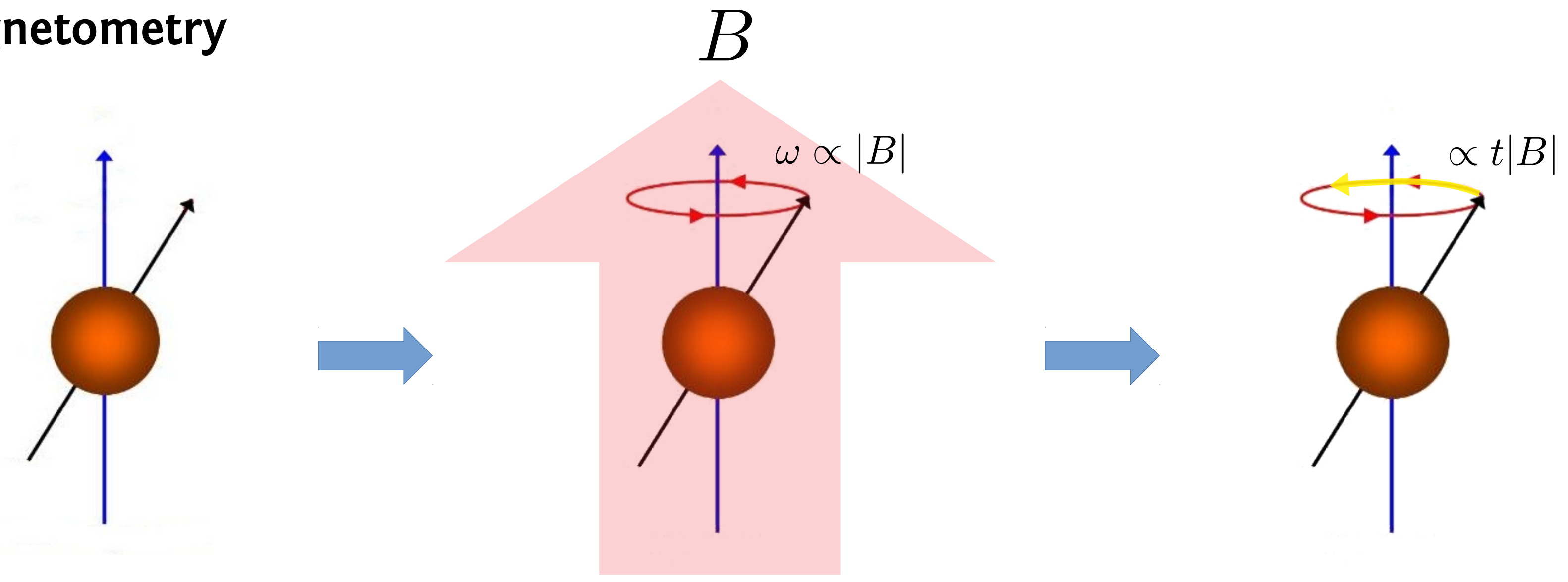

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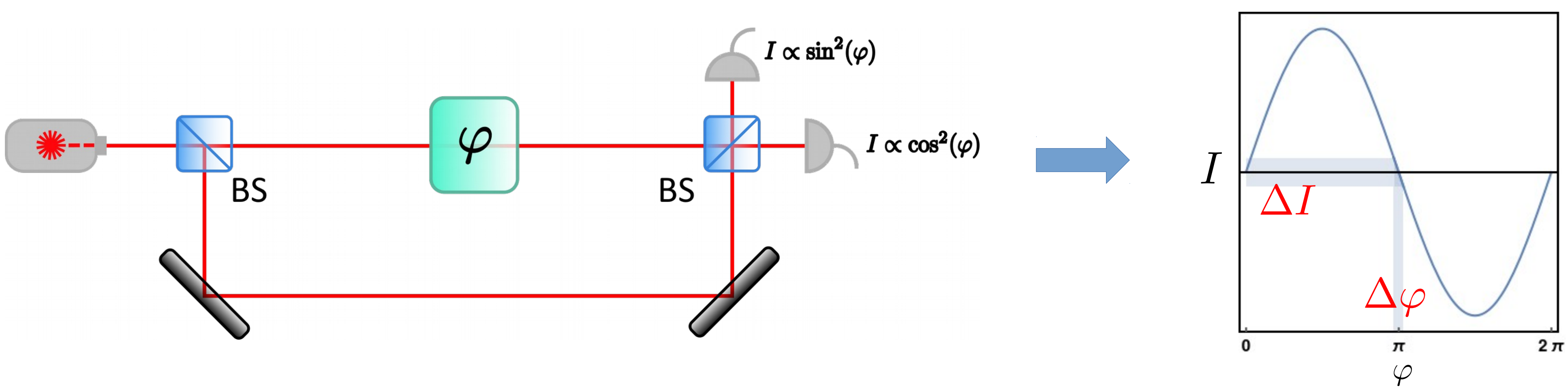
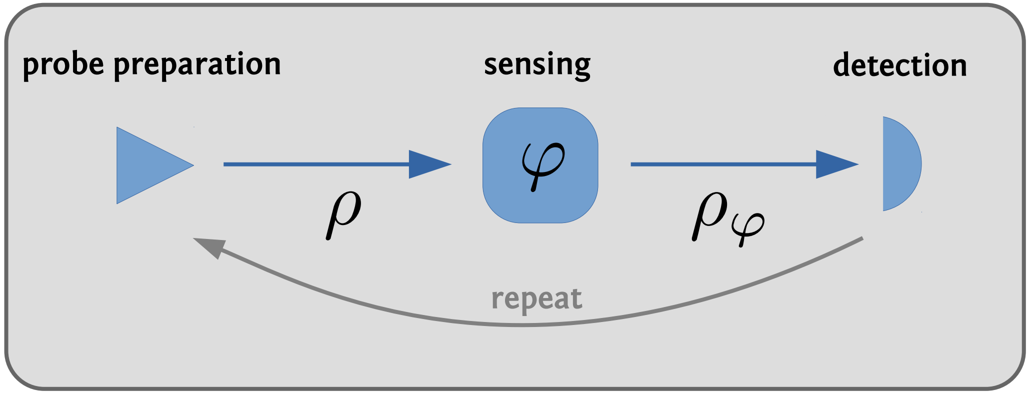
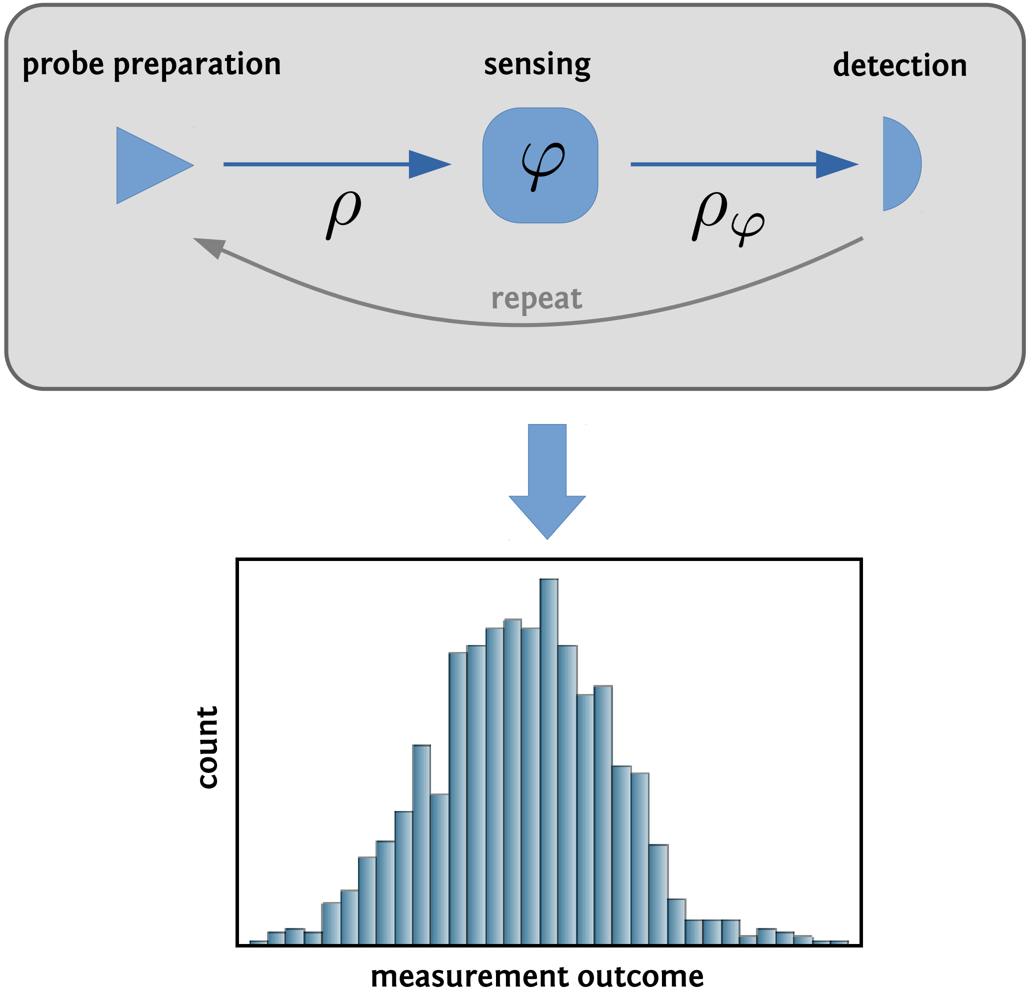


Single-parameter estimation



Measurements.

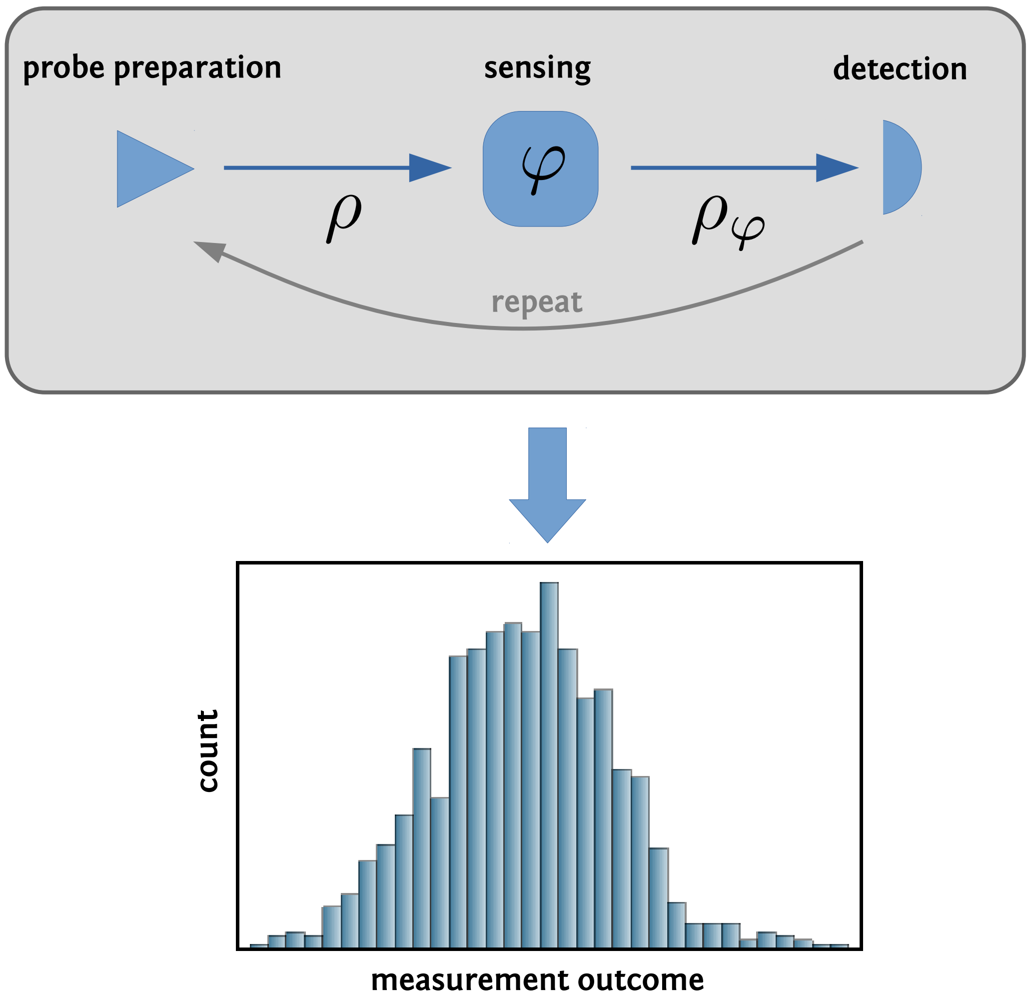
Single-parameter estimation



Measurements.

Raw data.

Single-parameter estimation



Measurements.

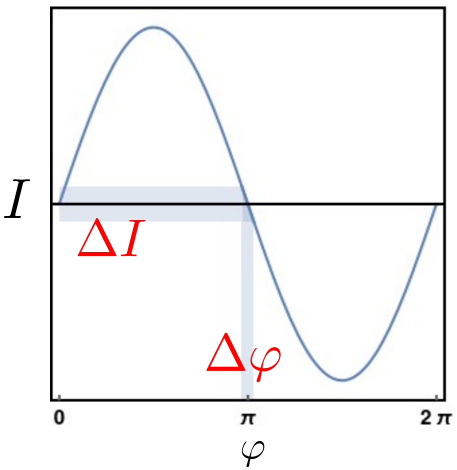
Raw data.

$$\hat{\varphi} = f(x_0, x_1 \ldots)$$

Estimation (post processing).

Focus on local estimation

- Sensitivity to small changes.
- Asymptotic scaling of the achievable precision.



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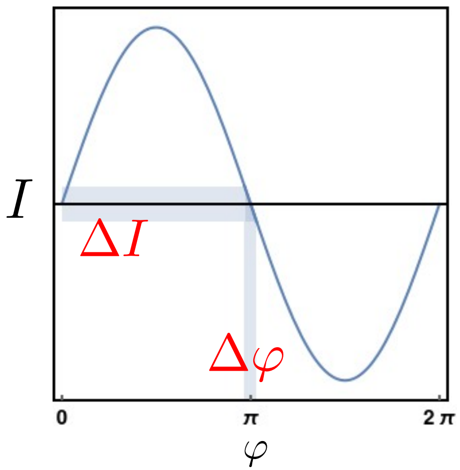


Classical vs. Quantum

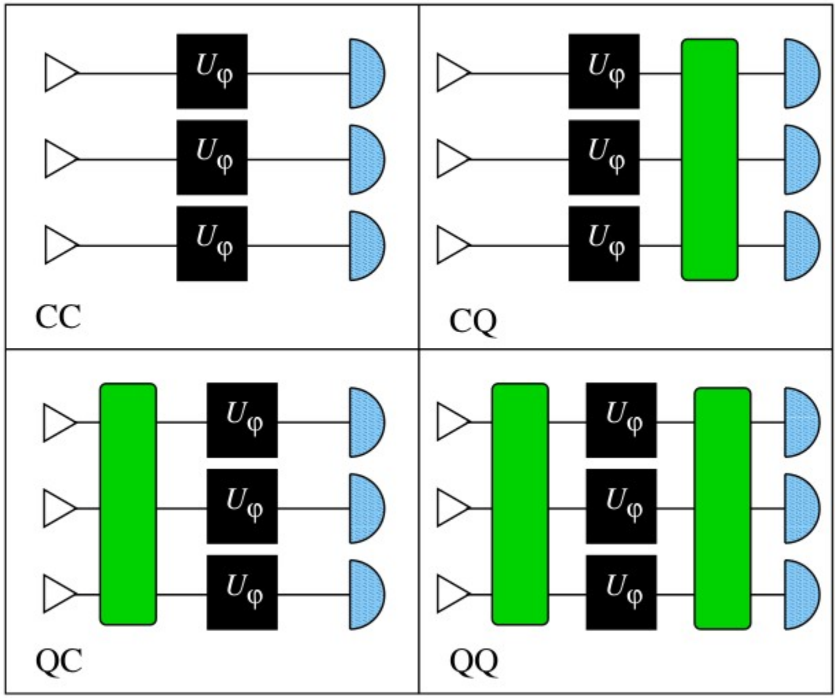
- Compare precision for fixed resources:

N size of probe used.

T total measurement time.

ν number of repetitions.

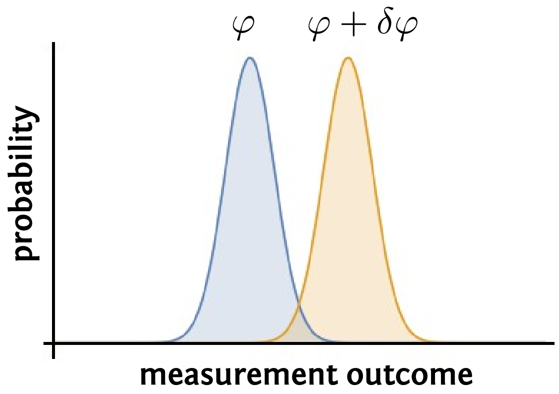
$$\Delta^2\varphi \sim ?(N, T, \nu)$$



Giovannetti, Lloyd, Maccone, *Phys. Rev. Lett.* 96, 010401 (2006).

Quantifying sensitivity - Fisher information

very sensitive...



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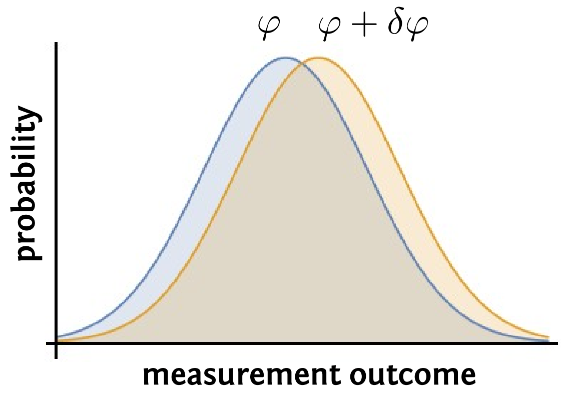

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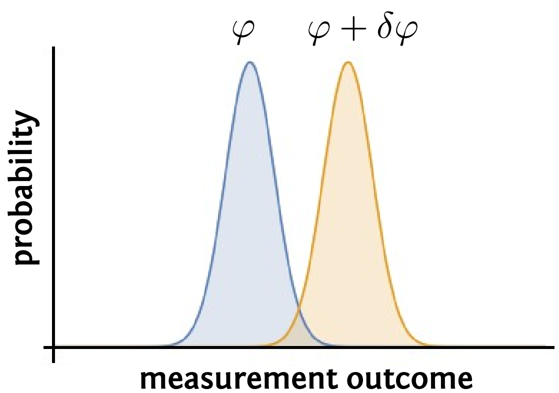

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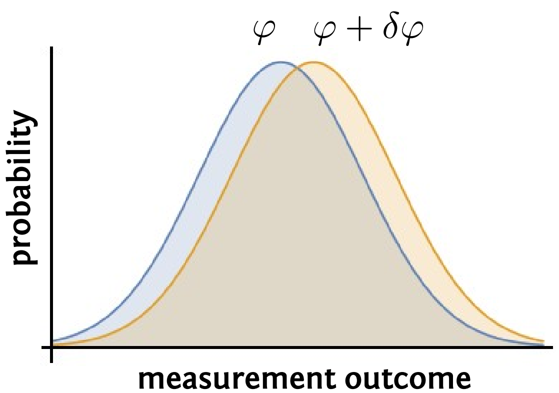

Fisher information – measures information about the parameter in the distribution

$$\mathcal{F}_\varphi = \mathcal{F}(p_\varphi) = \sum_x \left(\frac{\partial}{\partial \varphi} \log p_\varphi(x) \right)^2 p_\varphi(x)$$

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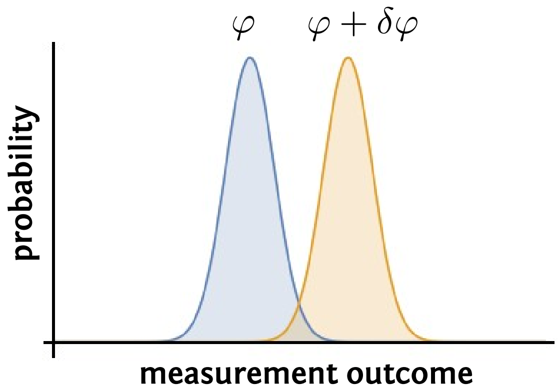

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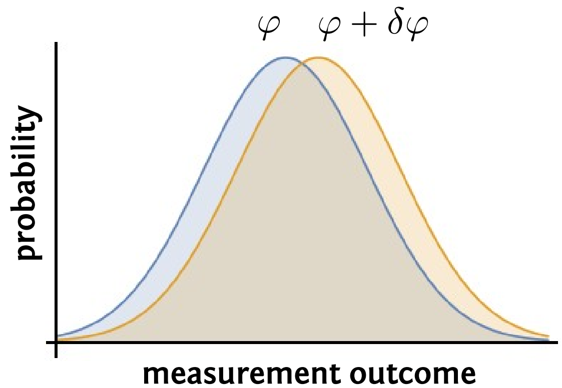

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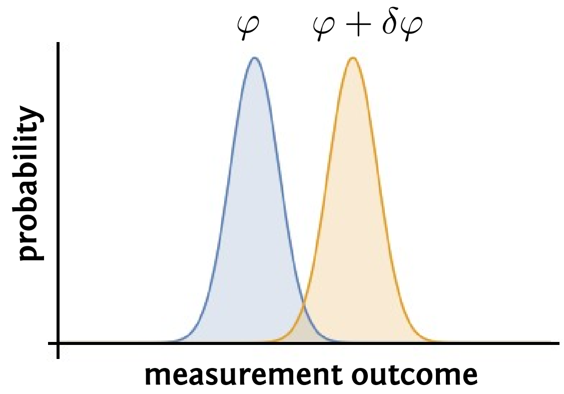

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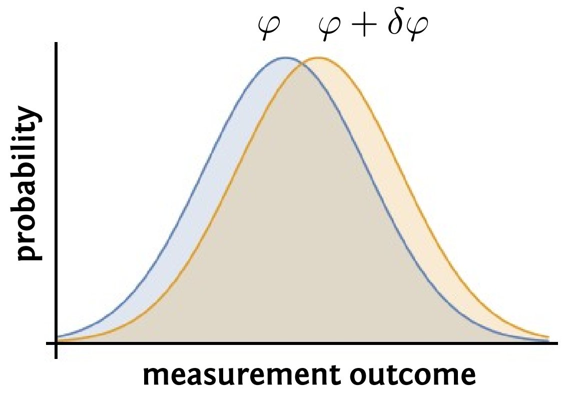

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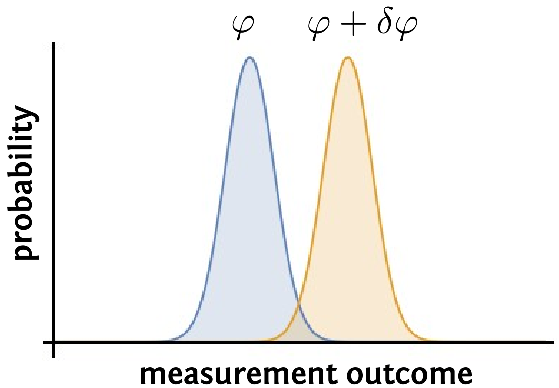

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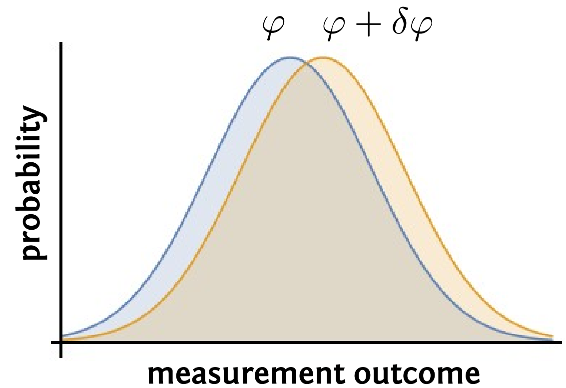

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Bounding estimation error – Cramér-Rao bound

$$\Delta^2\hat{\varphi} \geq \frac{1}{\mathcal{F}_\varphi}$$

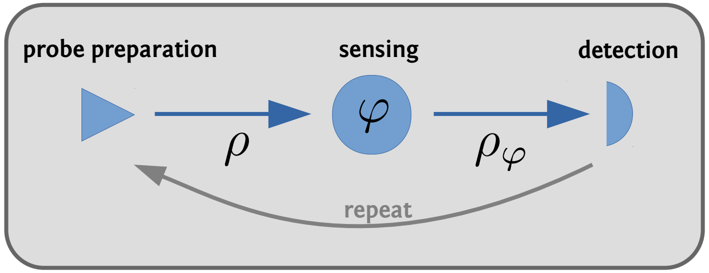
- Valid for any unbiased estimator.
- Can be saturated in the limit of many repetitions (and often before).

Harald Cramér

C. R. Rao

Quantum Fisher information

The Fisher information depends on the state and measurement.



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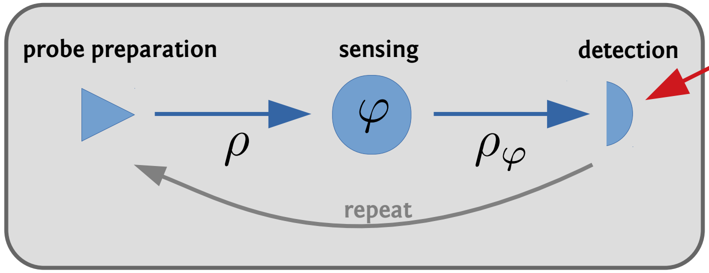

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$$p_\varphi(x) = \mathrm{Tr}\left[\Pi_x \rho_\varphi\right]$$

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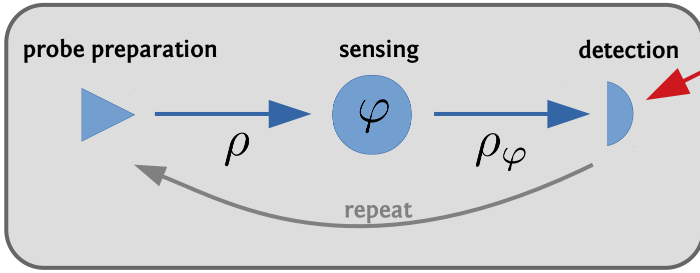

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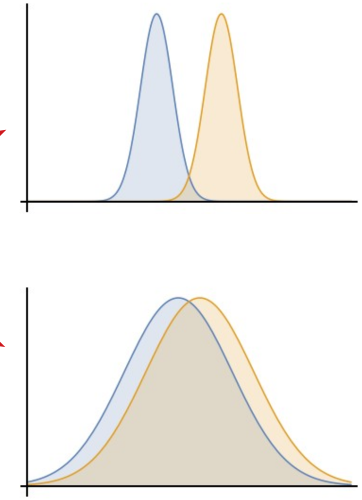

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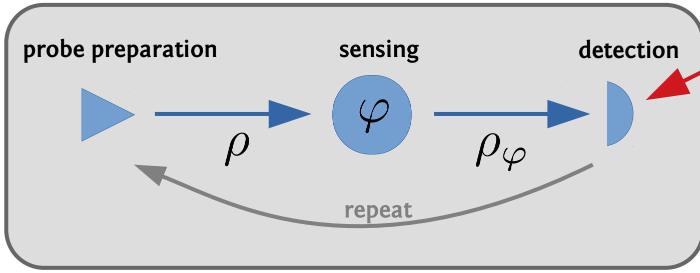

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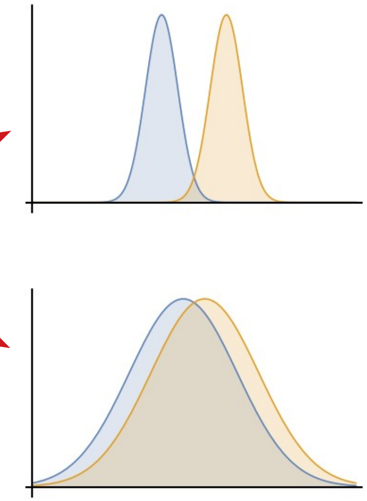

The *Quantum Fisher Information (QFI)* corresponds to the best possible measurement

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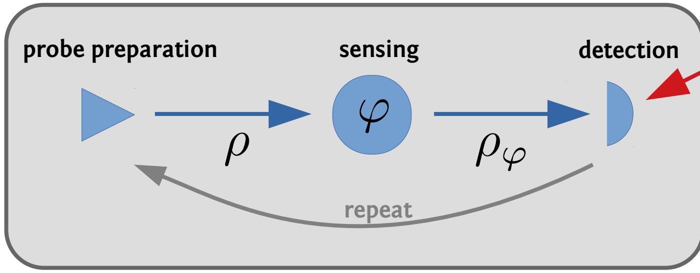

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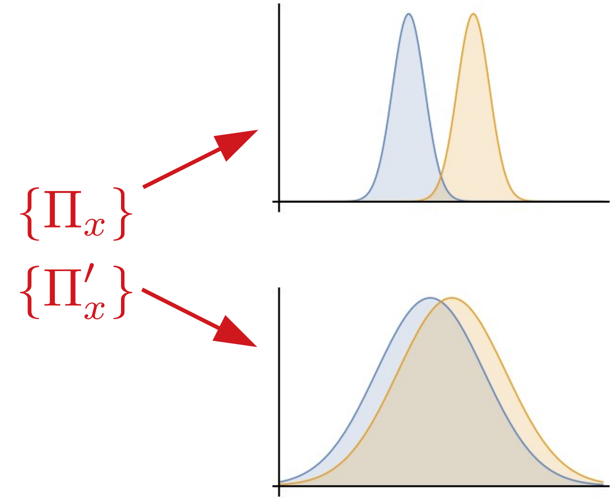

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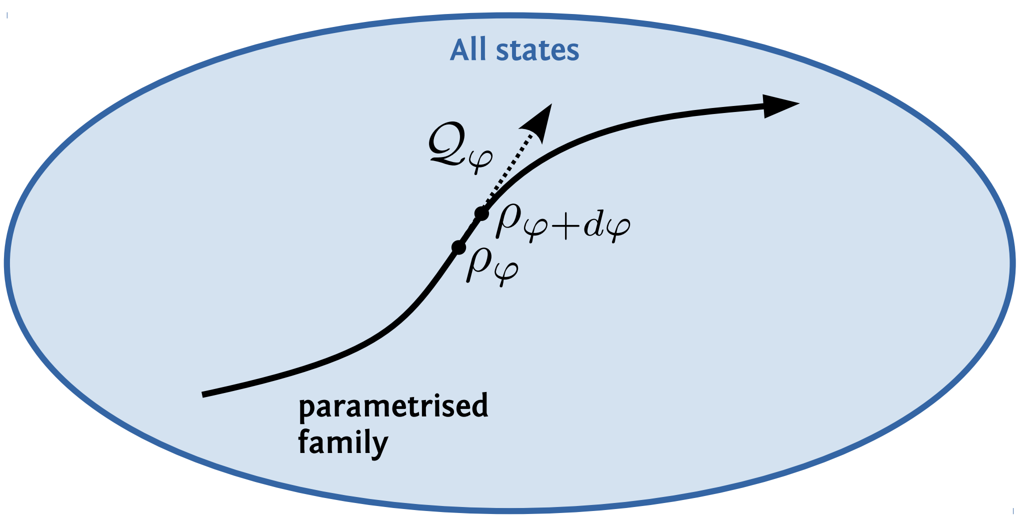

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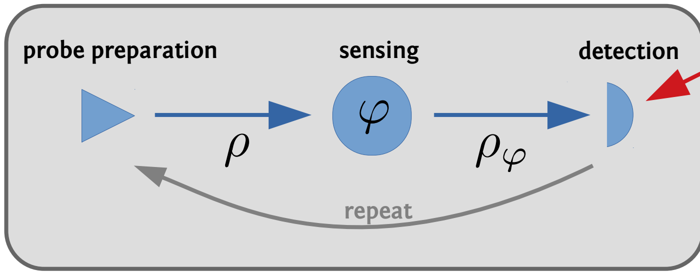

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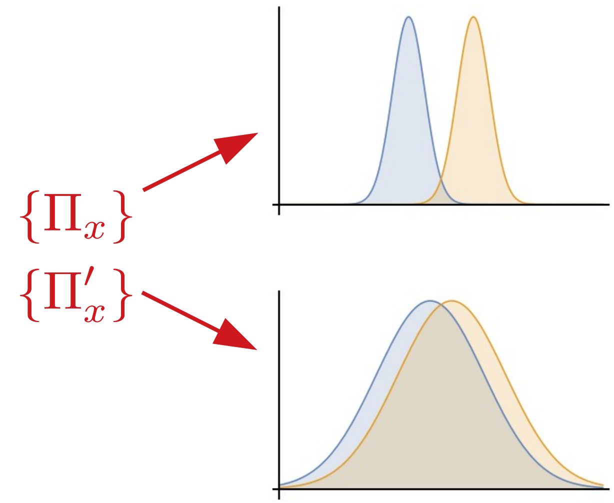

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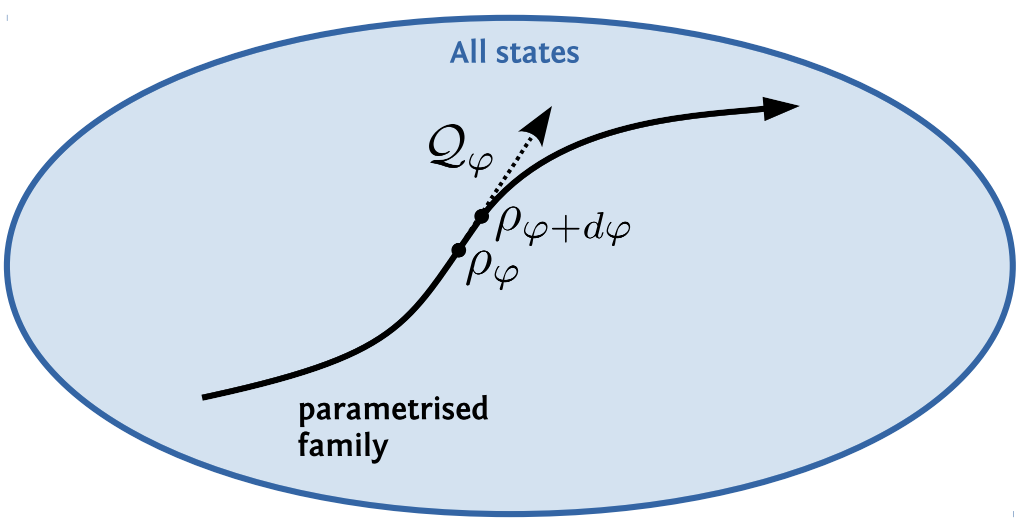

Quantum Cramér-Rao bound

$$\Delta^2 \hat{\varphi} \geq \frac{1}{\mathcal{Q}_\varphi}$$

can be saturated if the optimal measurement
can be implemented.

Standard Quantum Limit (SQL)

The QFI is additive…

$$\mathcal{Q}(\rho_\varphi^{\otimes N}) = N\,\mathcal{Q}(\rho_\varphi)$$

information from independent trials simply adds

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parameter-independent mixing can only make things worse

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parameter-independent mixing can only make things worse

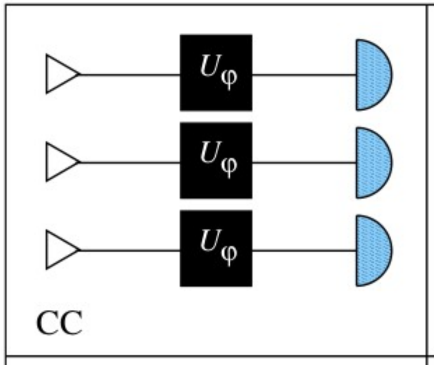
$\Downarrow$

Classically, the error scales linearly with probe size

$$\Delta^2\varphi \geq \frac{1}{N\mathcal{Q}_\varphi}$$

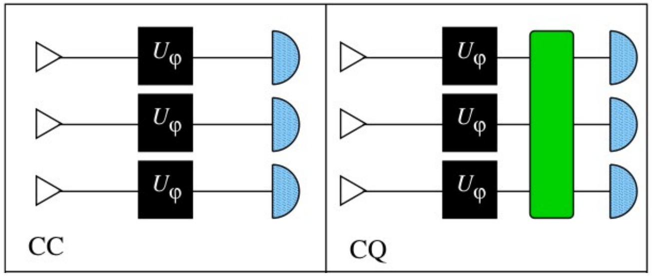
(w/o entanglement)

This linear scaling with *N* is the SQL.



Can we beat the SQL?

- For classical input, collective measurements do not help.



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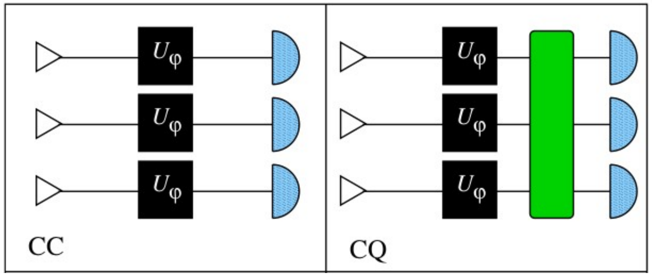
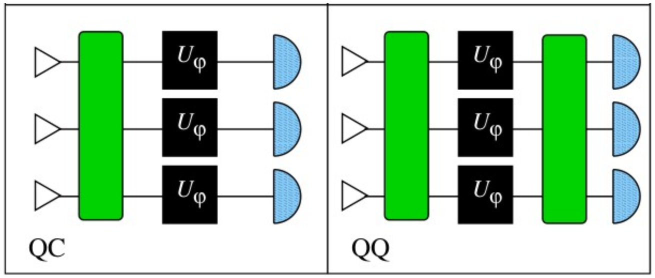


$$\Delta^2\varphi \geq \frac{1}{N Q_\varphi}$$

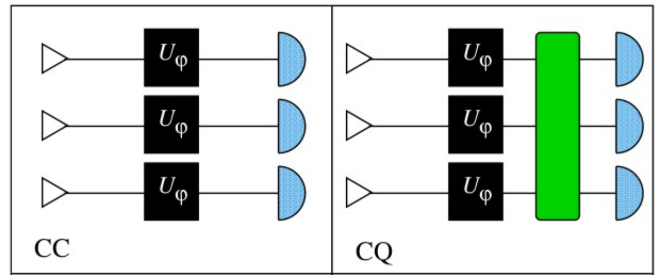
- Entangled states *do improve precision.* collective measurements are not required. xxx



$$\Delta^2\varphi < \frac{1}{N Q_\varphi}$$

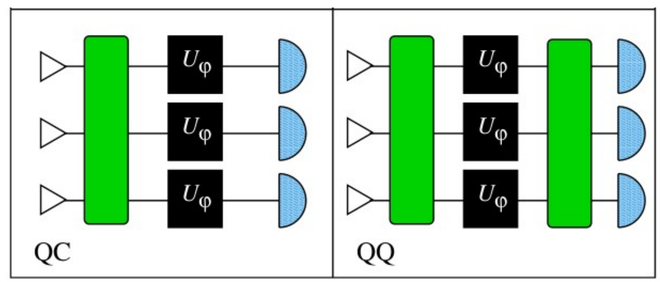
Can we beat the SQL?

- For classical input, collective measurements do not help.



$$\Delta^2\varphi \geq \frac{1}{NQ_\varphi}$$

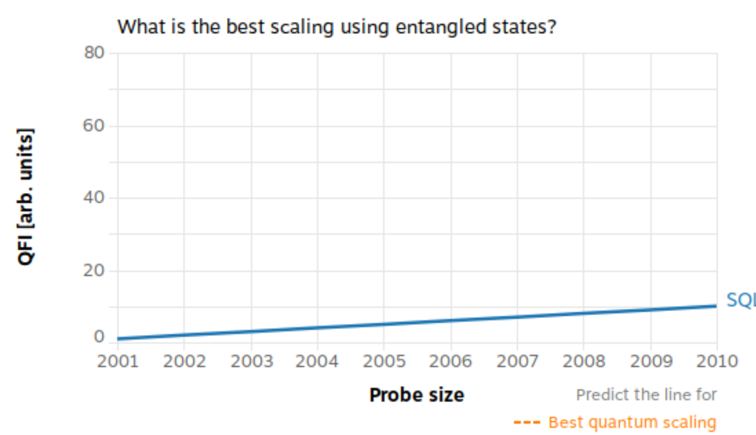
- Entangled states *do improve precision.* collective measurements are not required. xxx



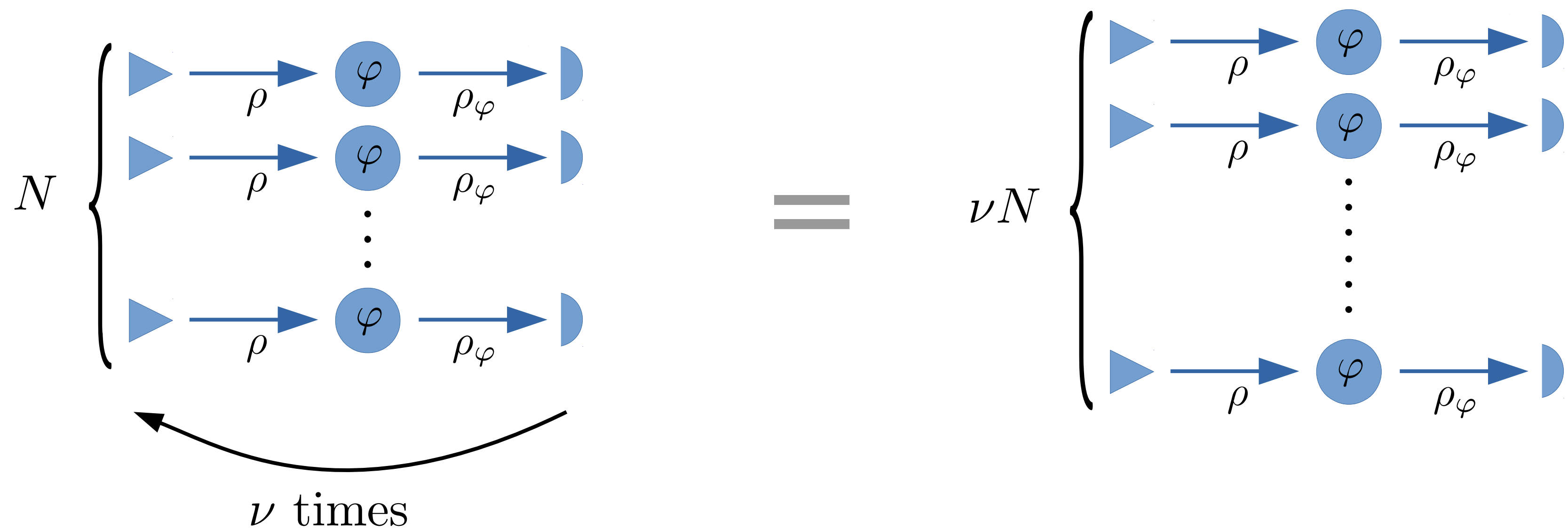
$$\Delta^2\varphi < \frac{1}{NQ_\varphi}$$

What do you think the best scaling is?

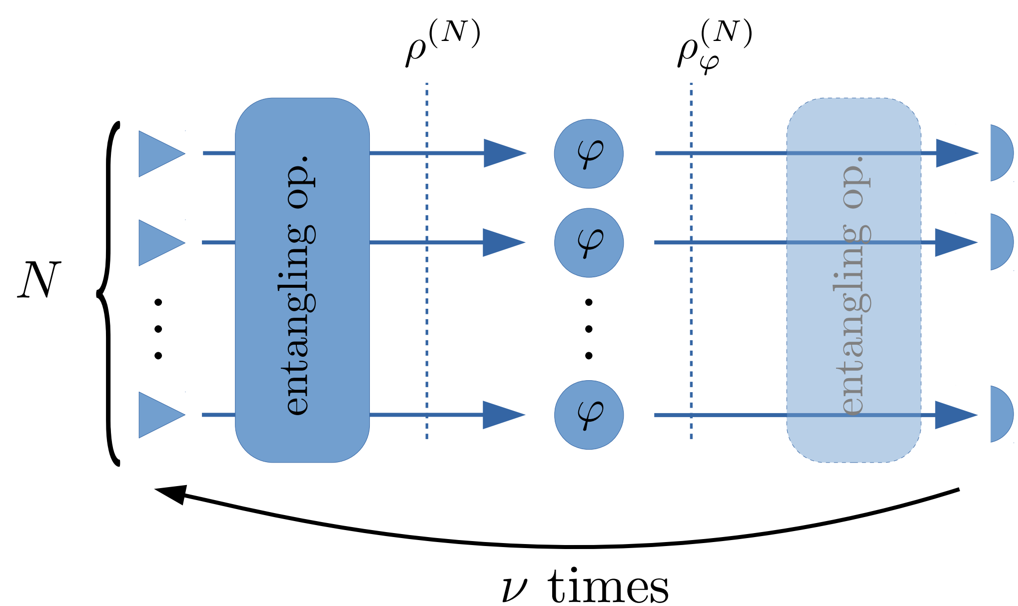
jonatanbohrbrask.dk/scaling



For independent probe particles



For entangled probes



The Heisenberg Limit

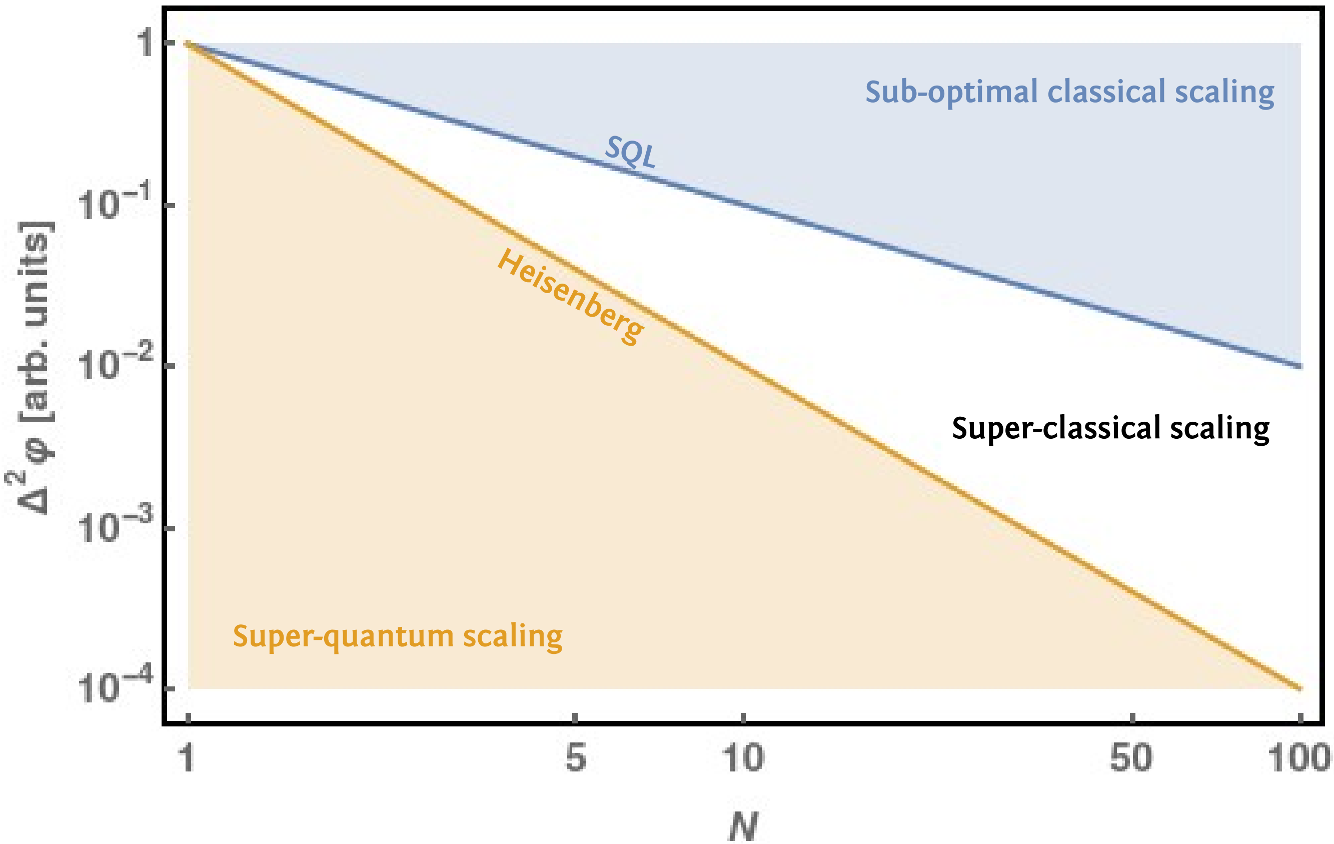
The best classical scaling

$$\Delta^2\varphi \propto \frac{1}{\nu N}$$

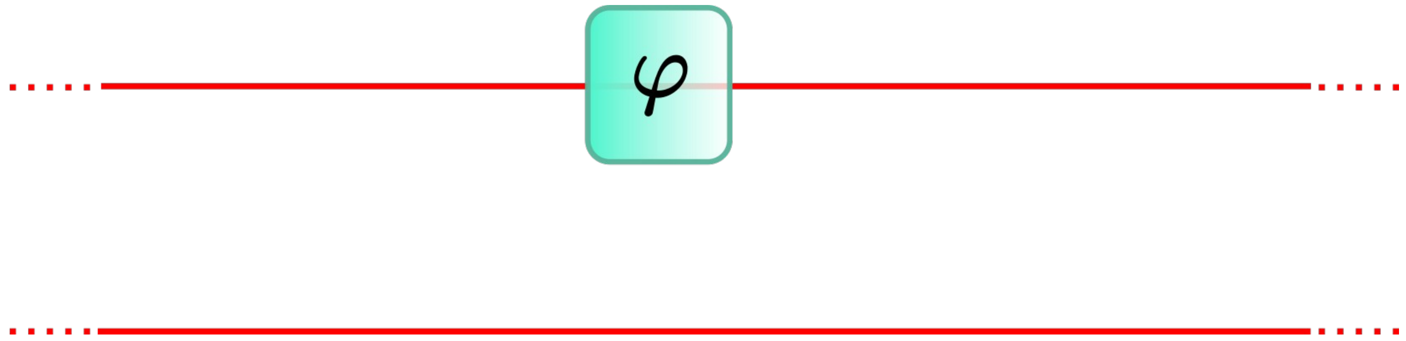
The best quantum scaling

$$\Delta^2\varphi \propto \frac{1}{\nu N^2}$$

On a log-log scale...



Achieving the Heisenberg limit – optical phase estimation



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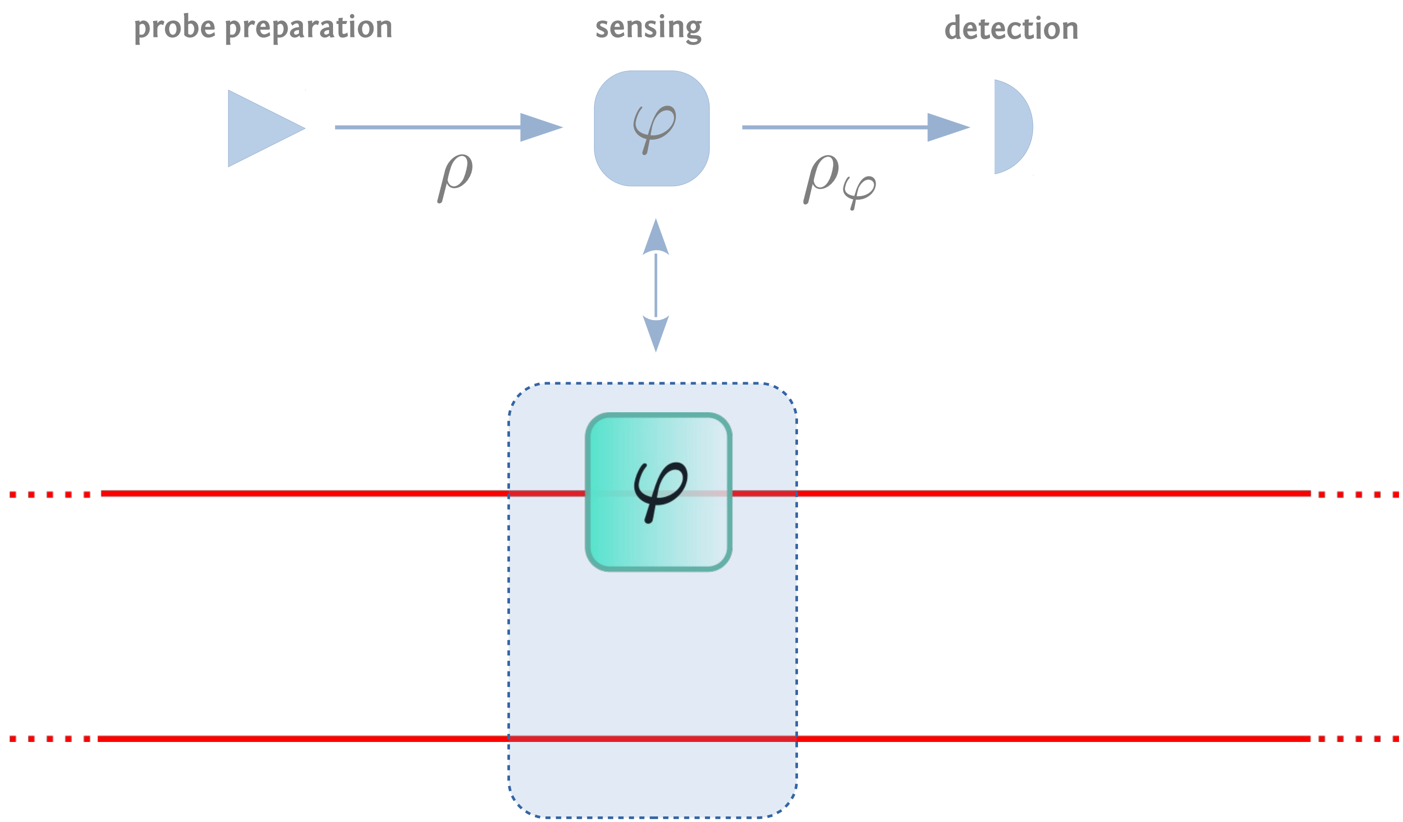
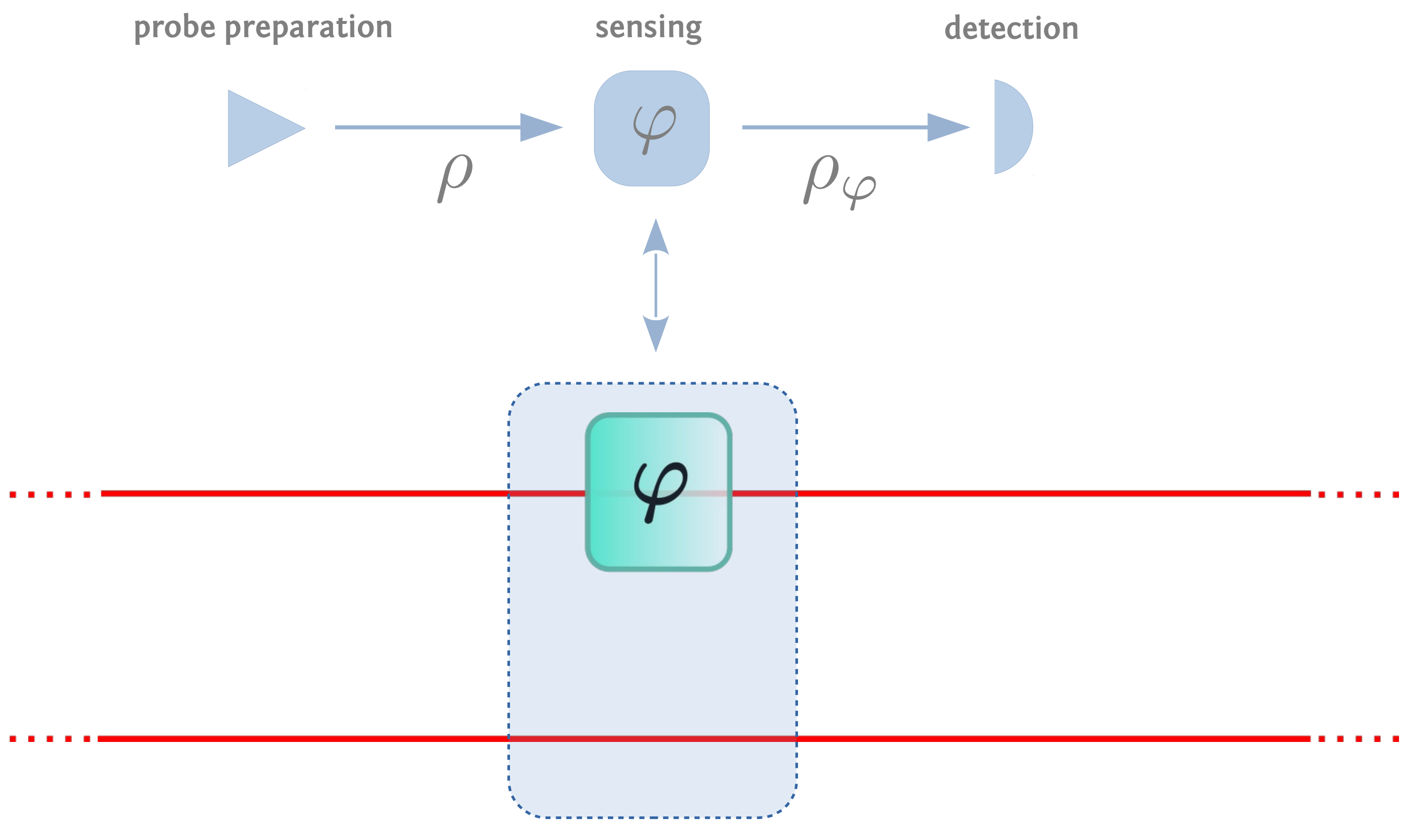
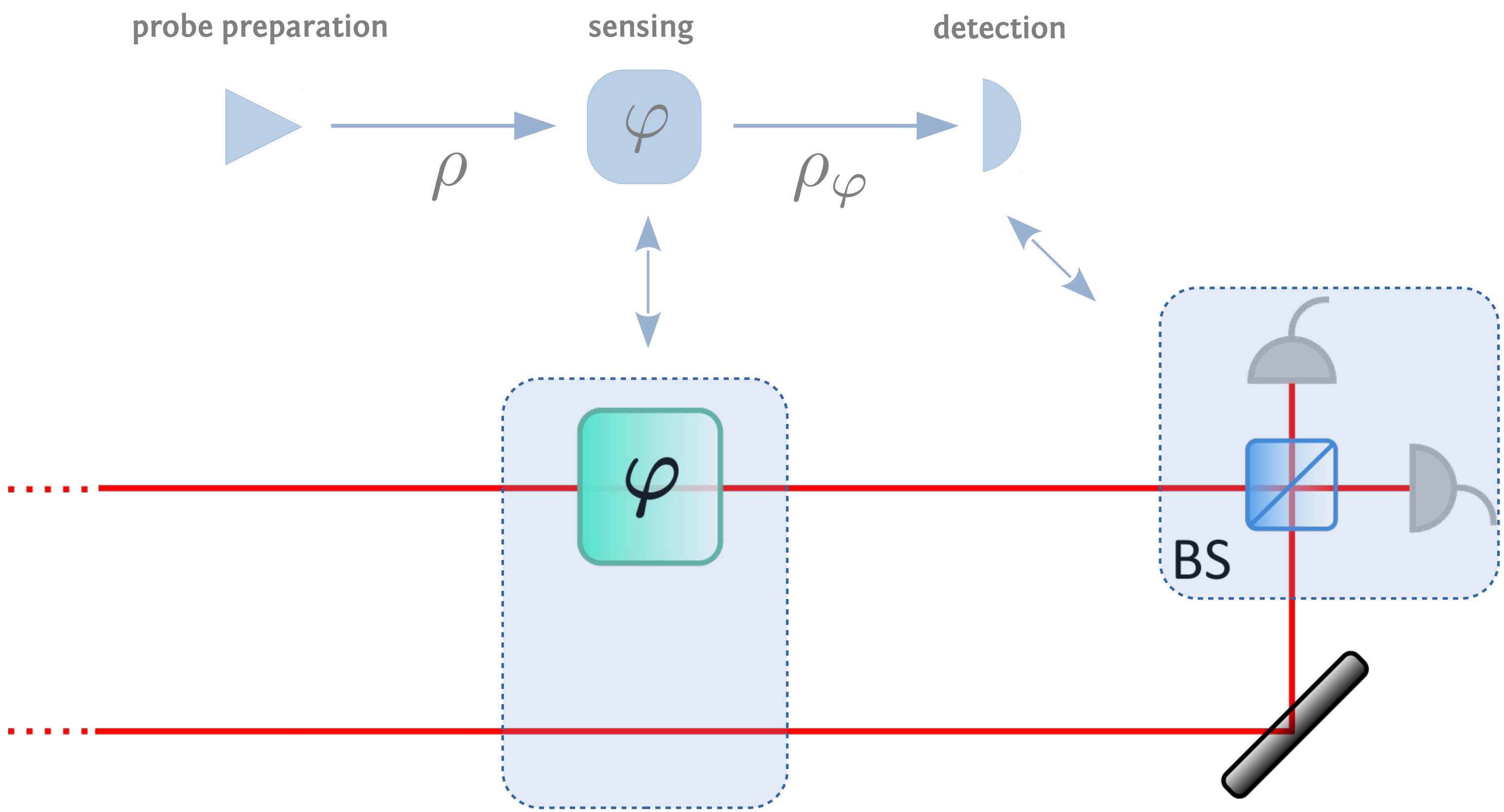


Achieving the Heisenberg limit – optical phase estimation



$$H \propto \hat{n}_A$$
$$U = e^{-iH\tau} = e^{i\varphi\,\hat{n}_A}$$

Achieving the Heisenberg limit – optical phase estimation



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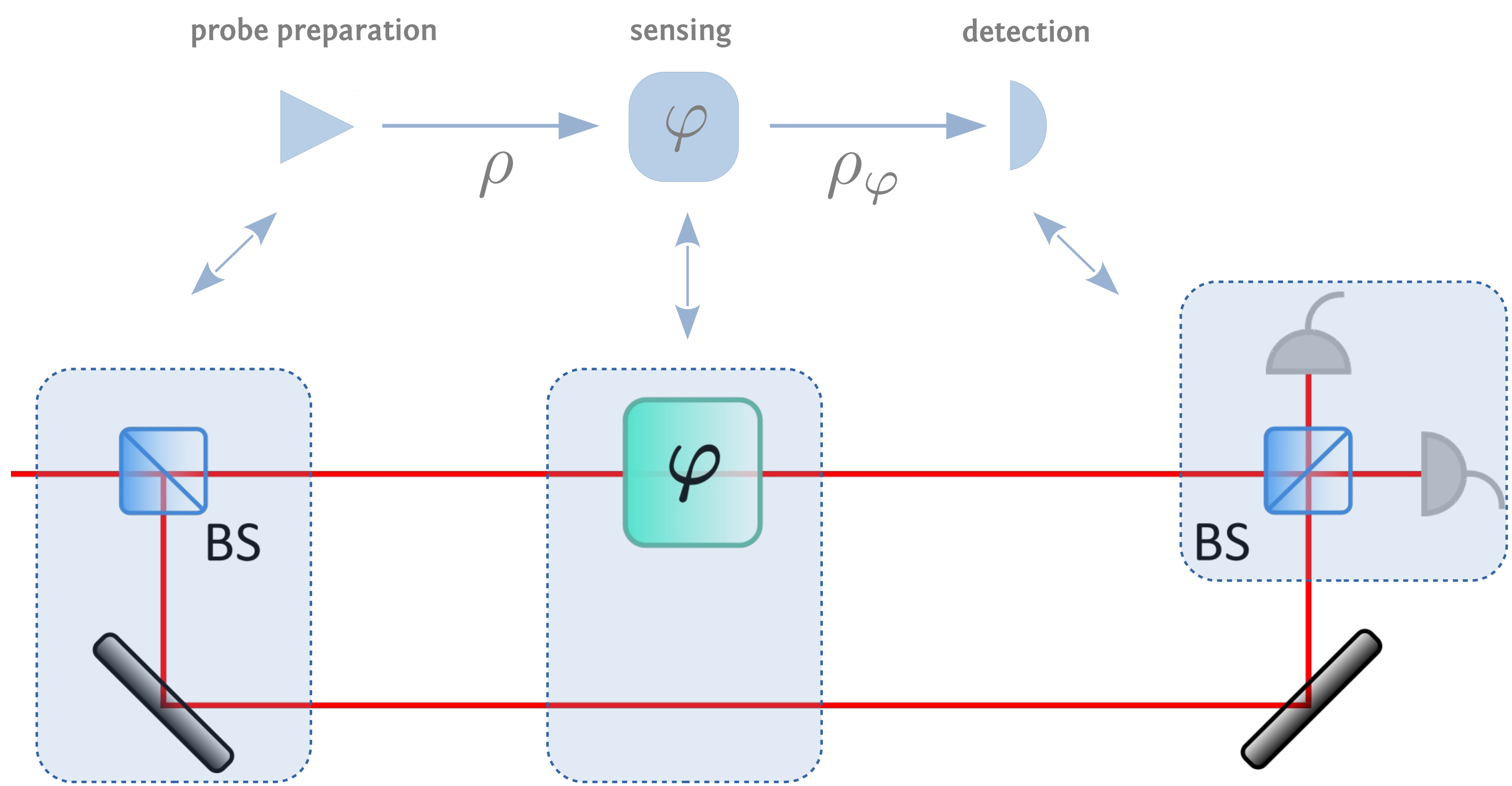

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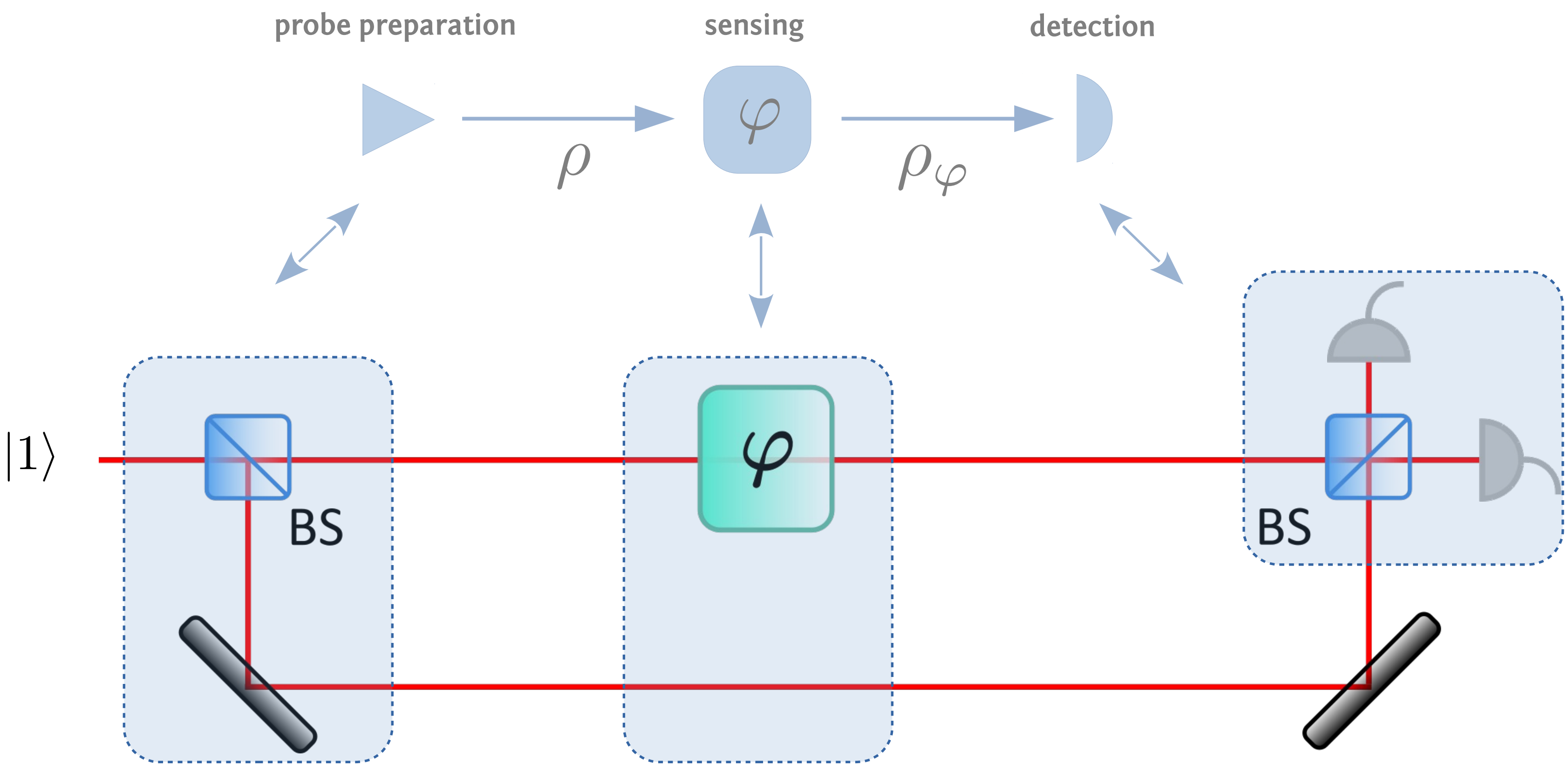

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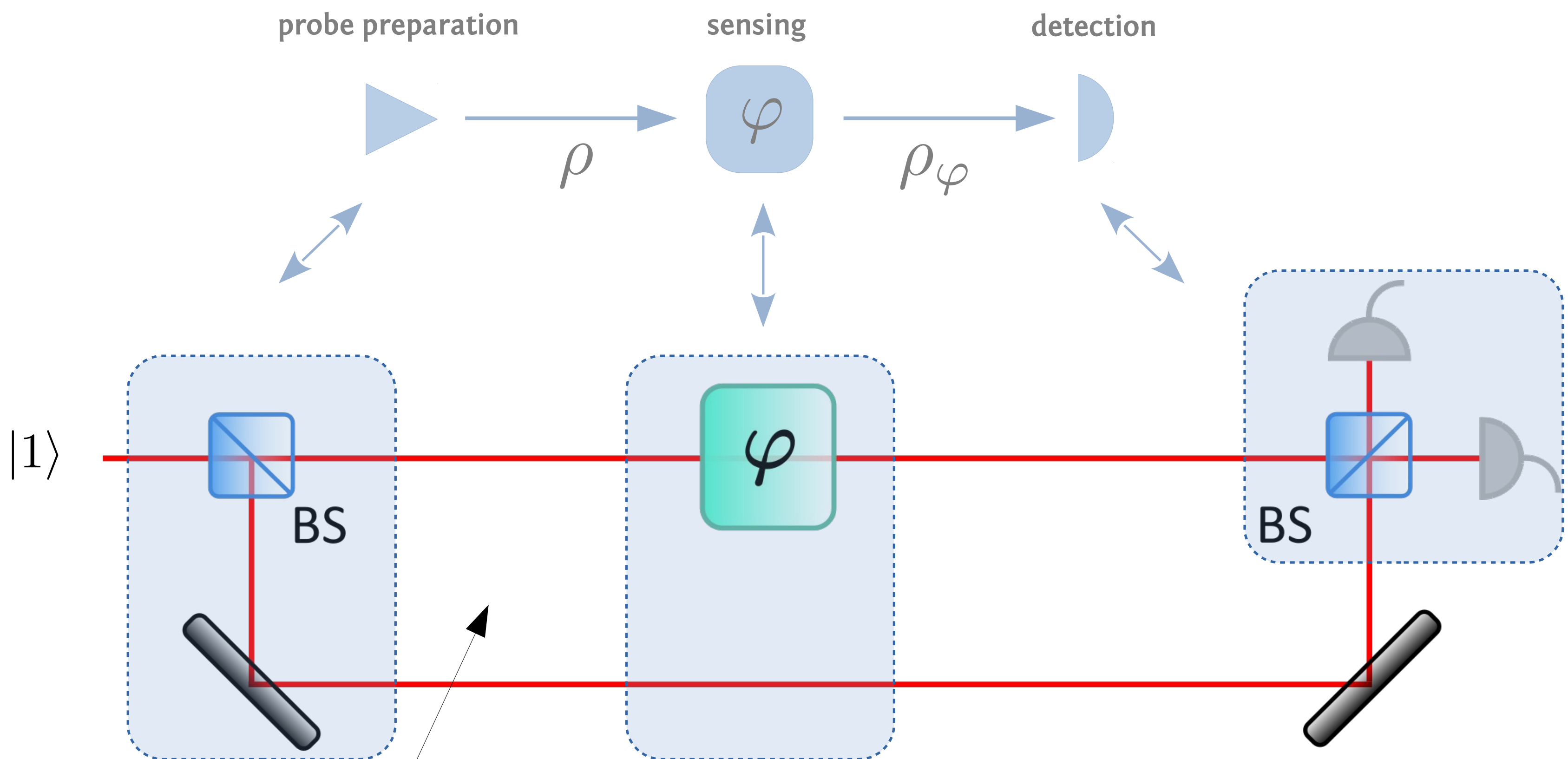

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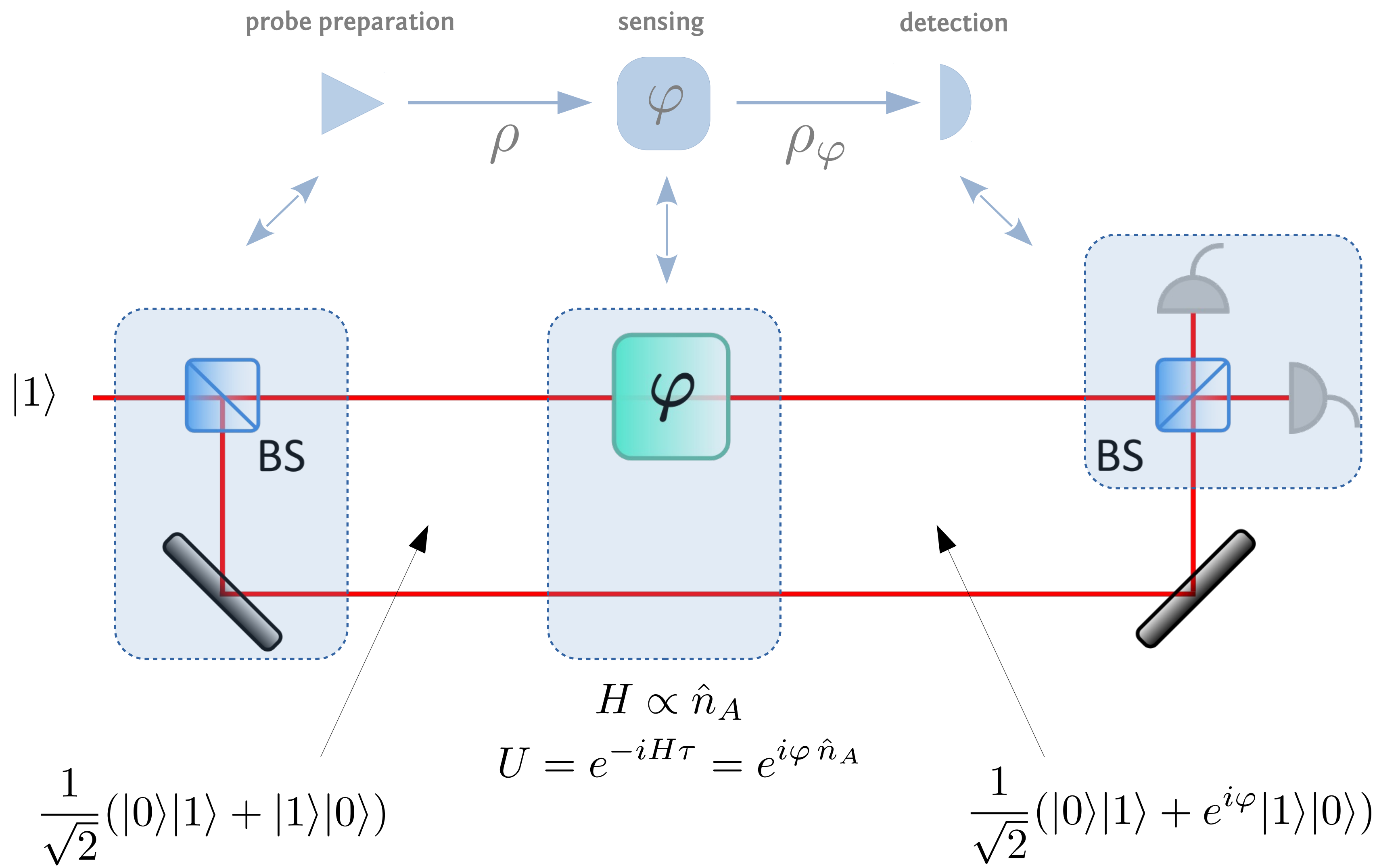

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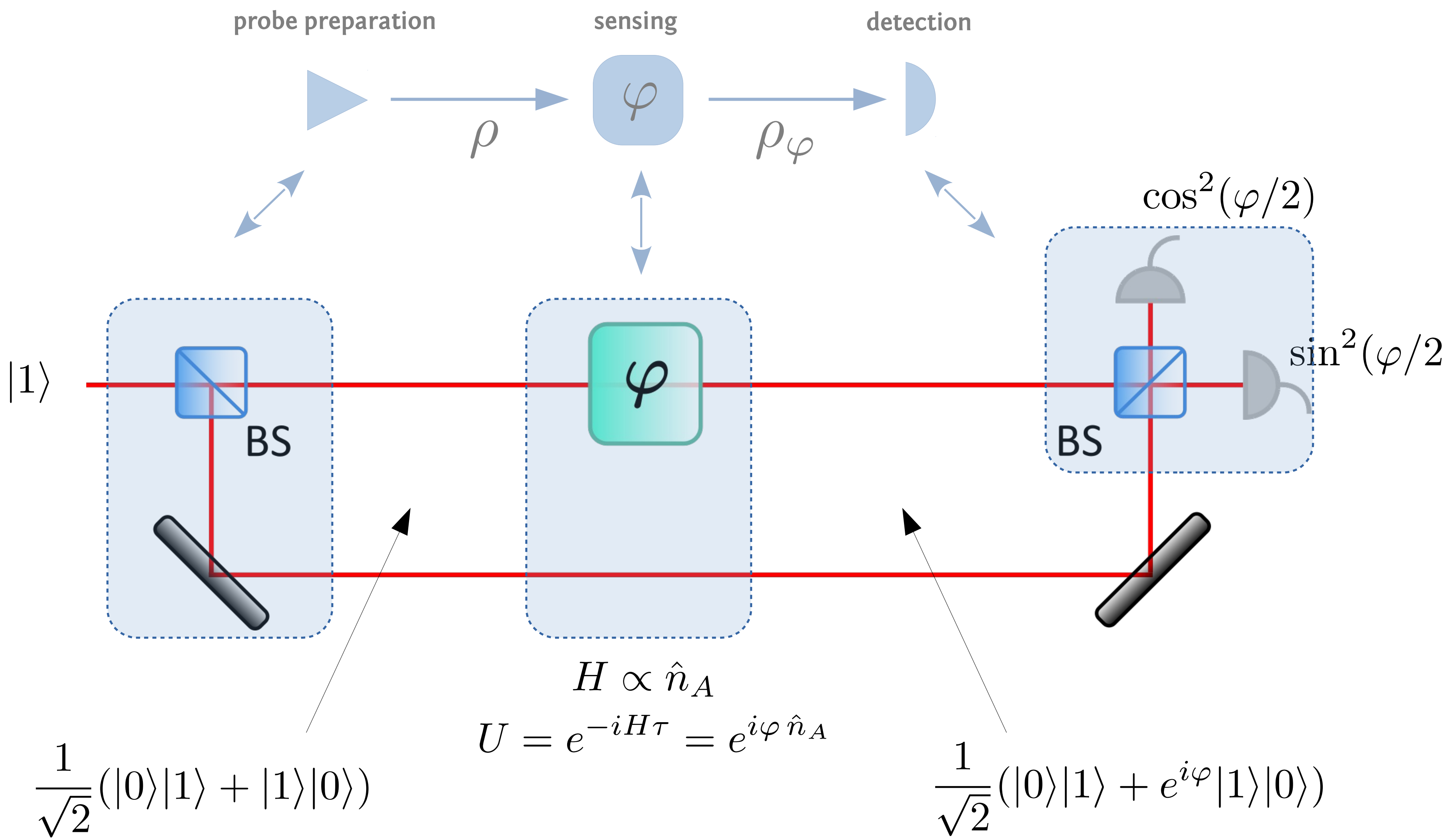

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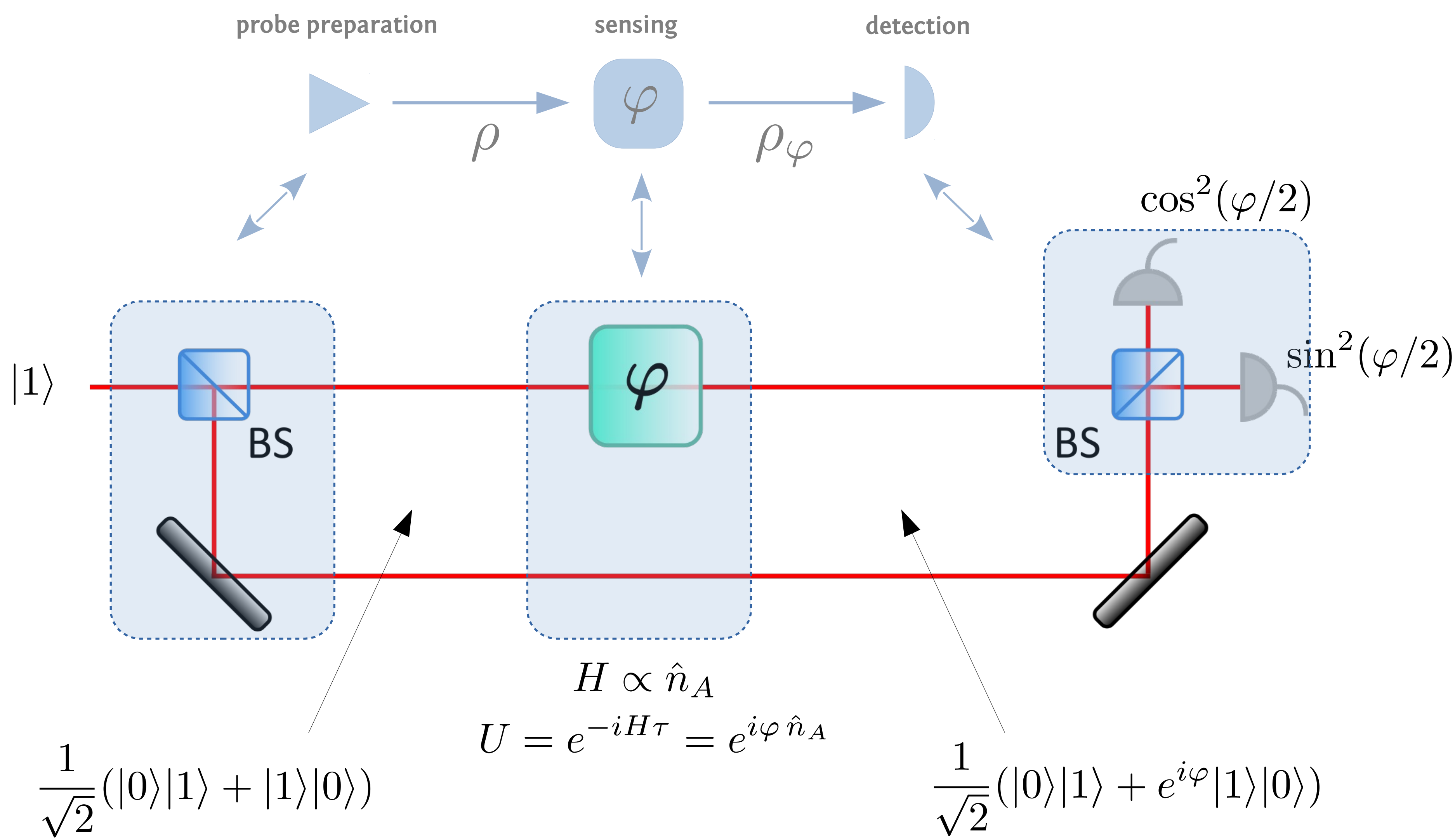

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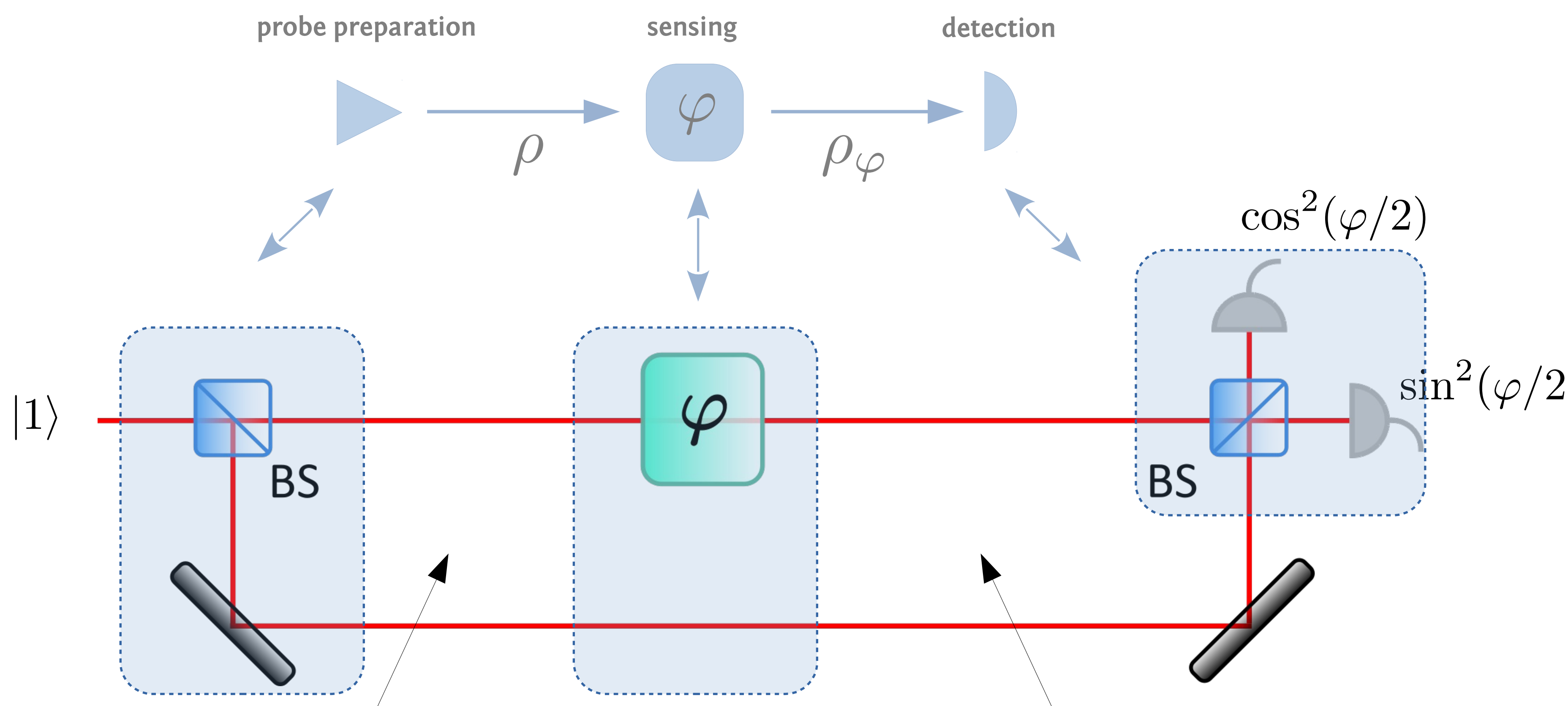

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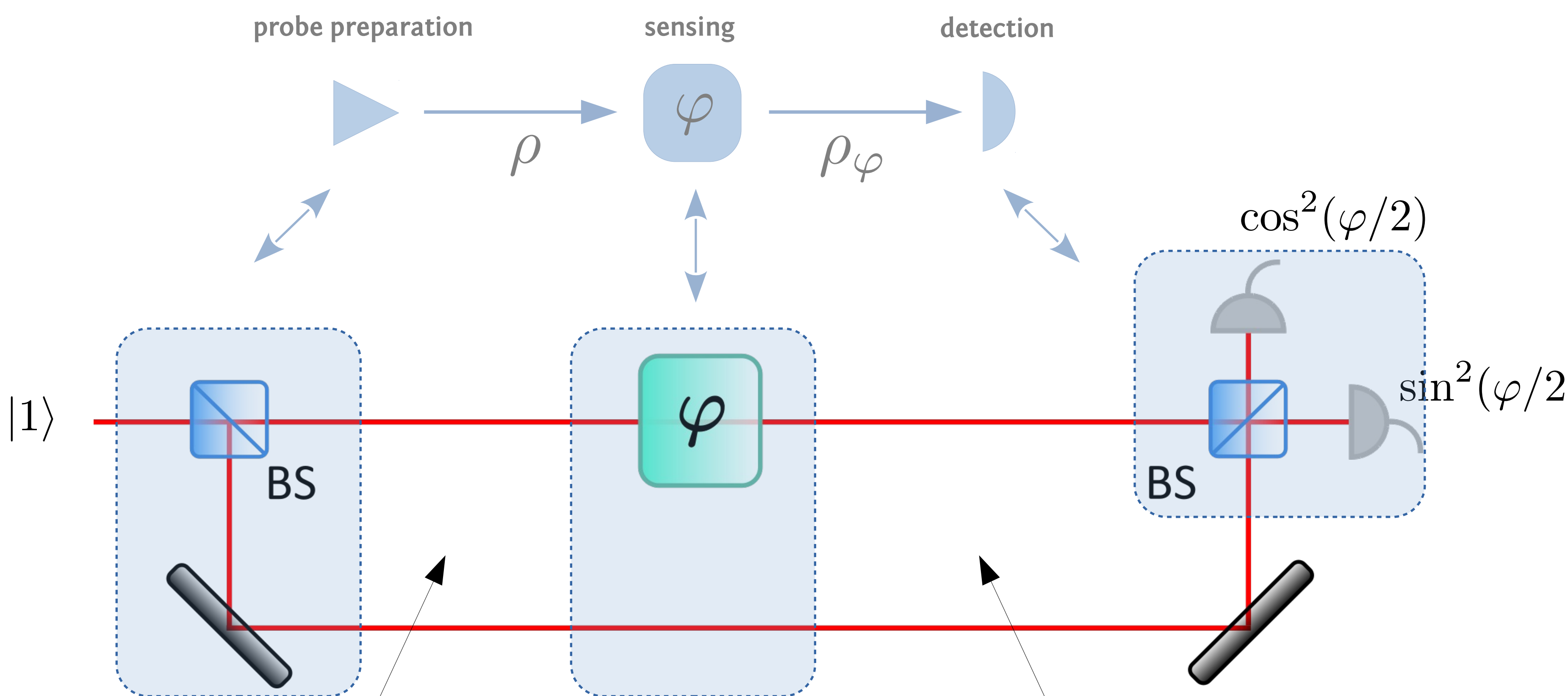
Classical strategy

$$\mathcal{F}_\varphi = 1$$

Achieving the Heisenberg limit – optical phase estimation



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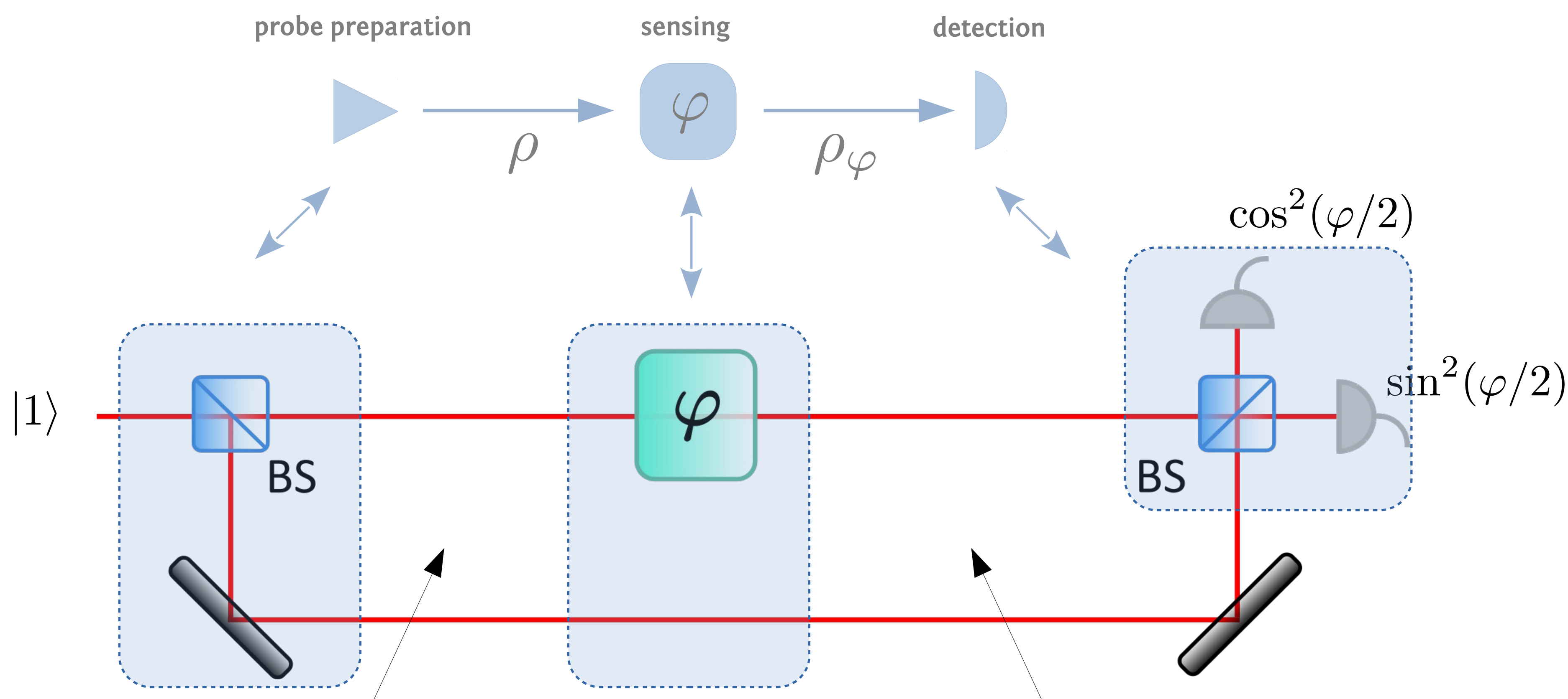

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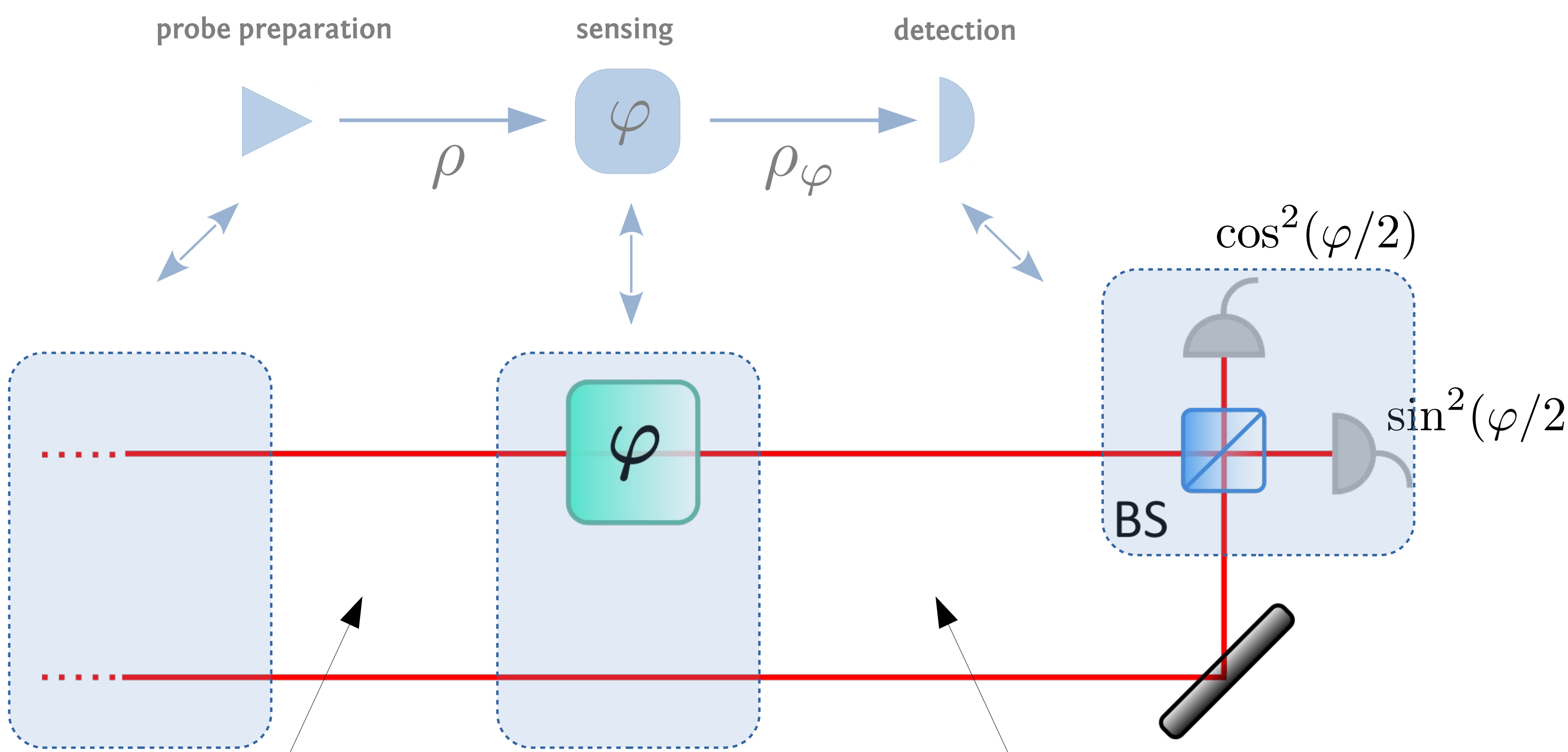

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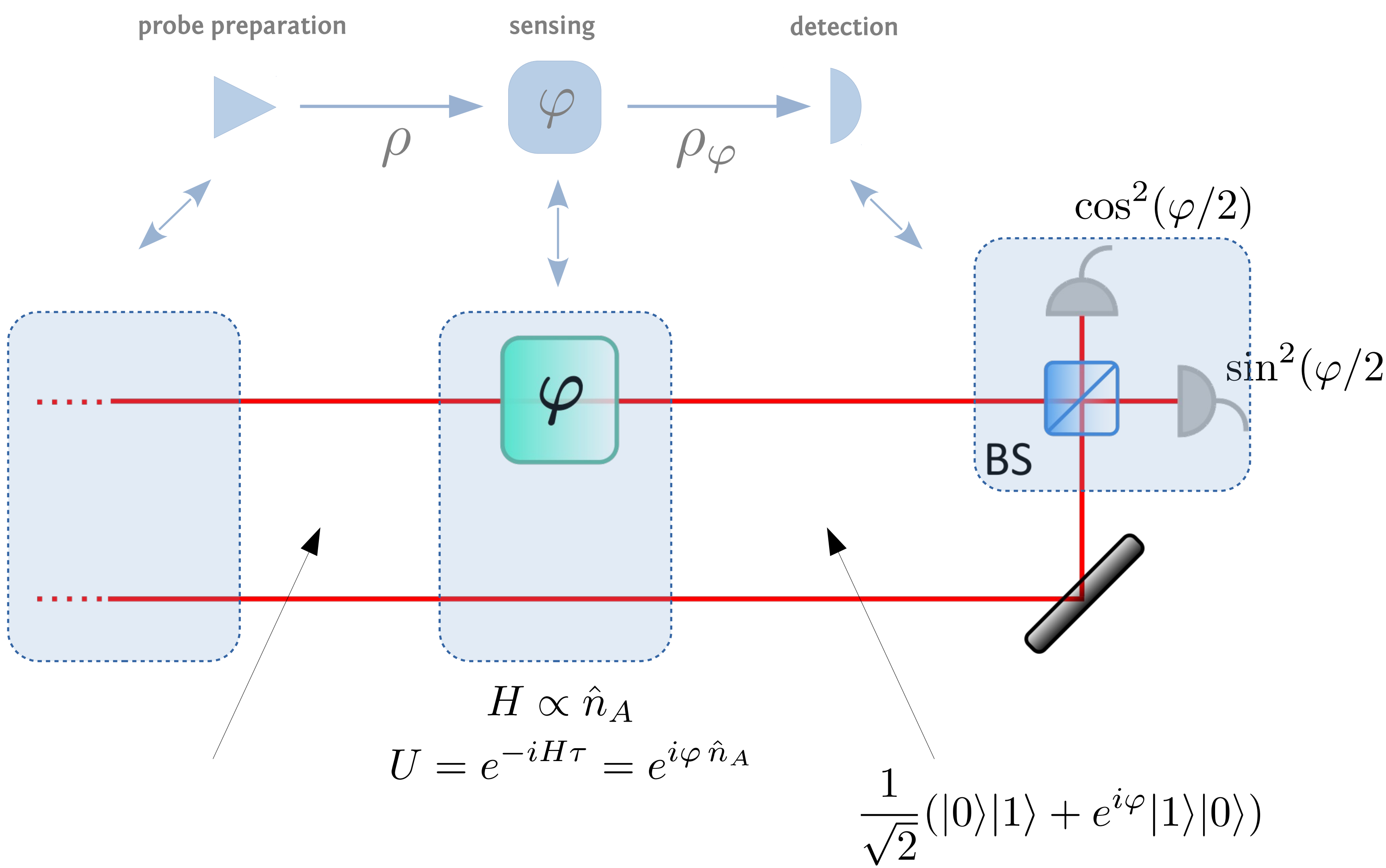

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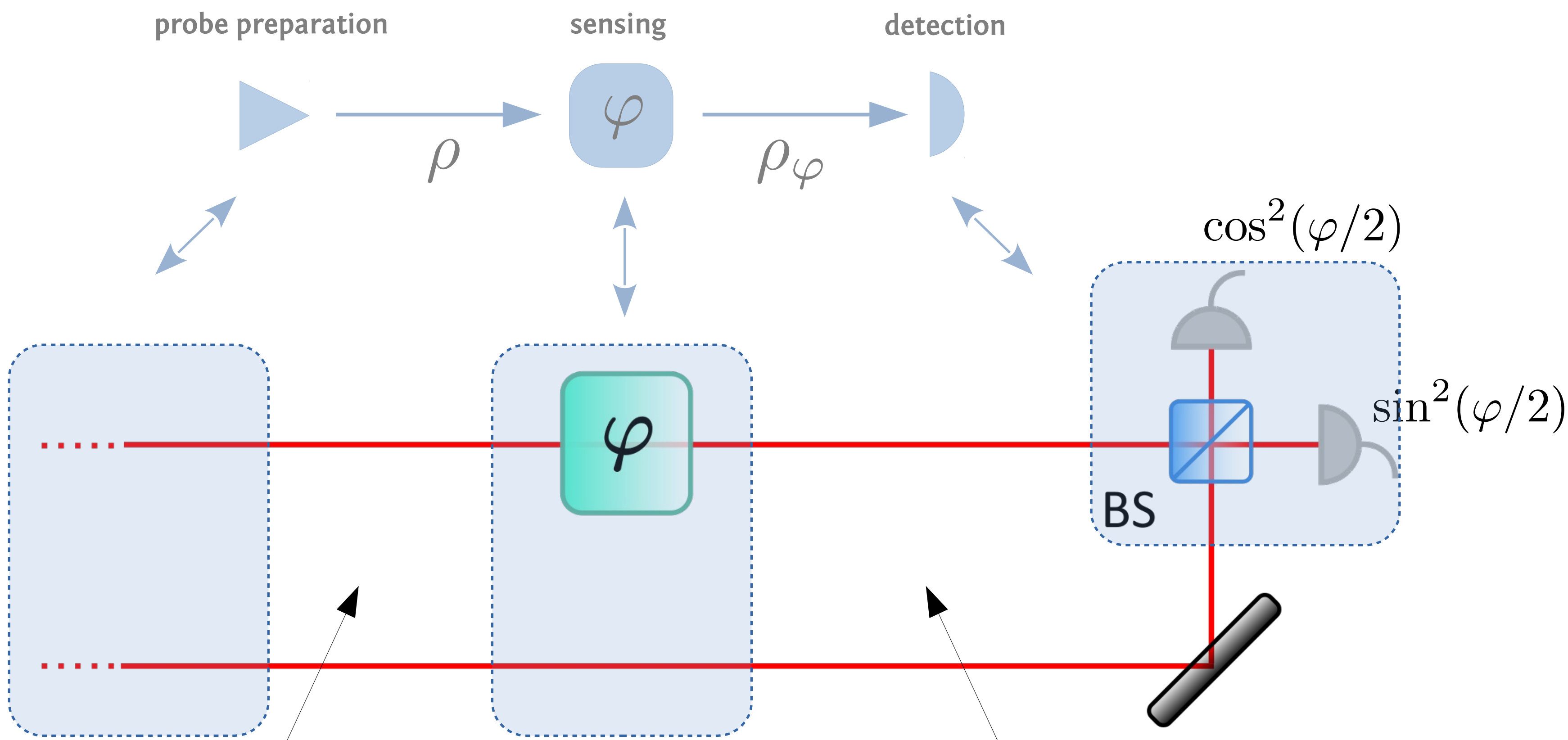

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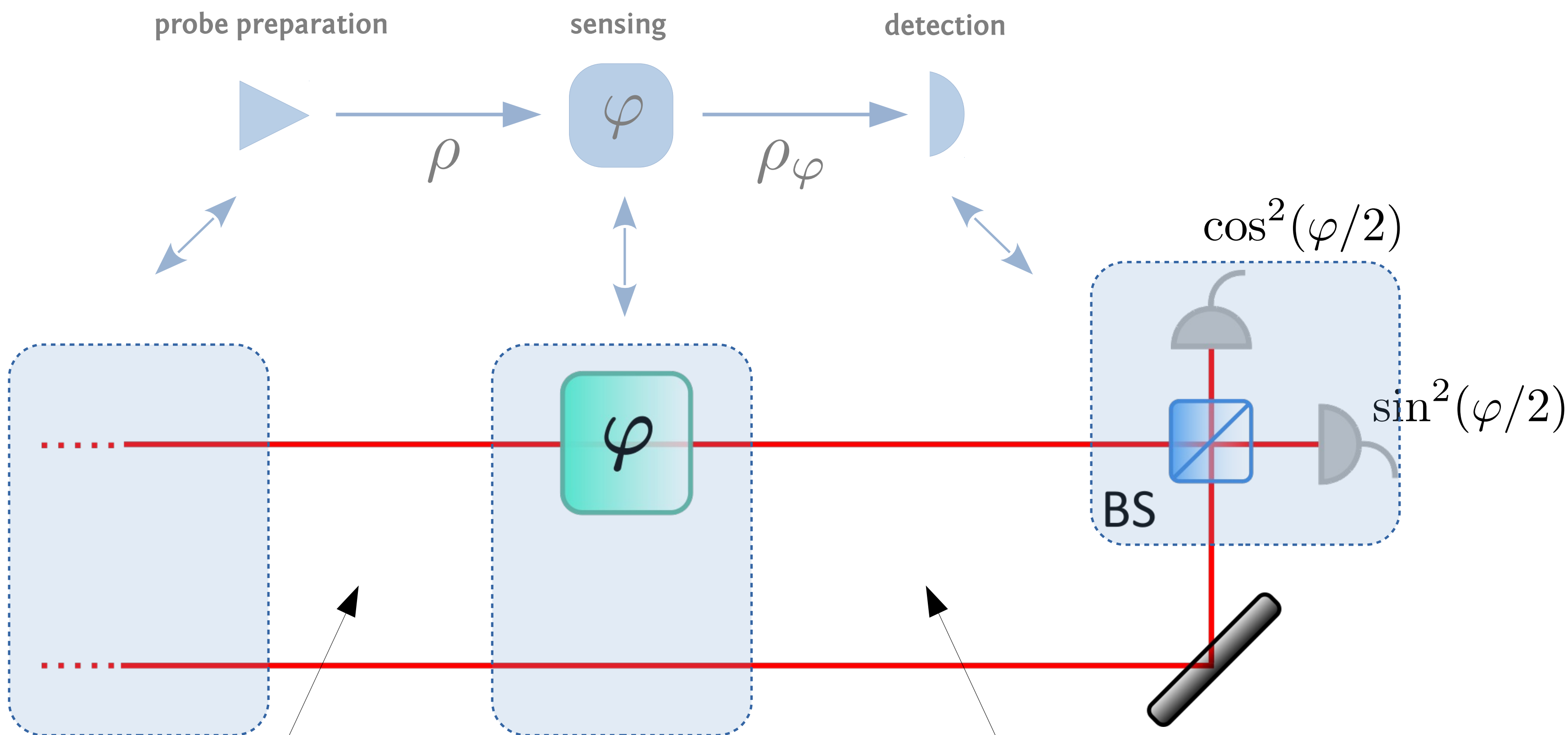

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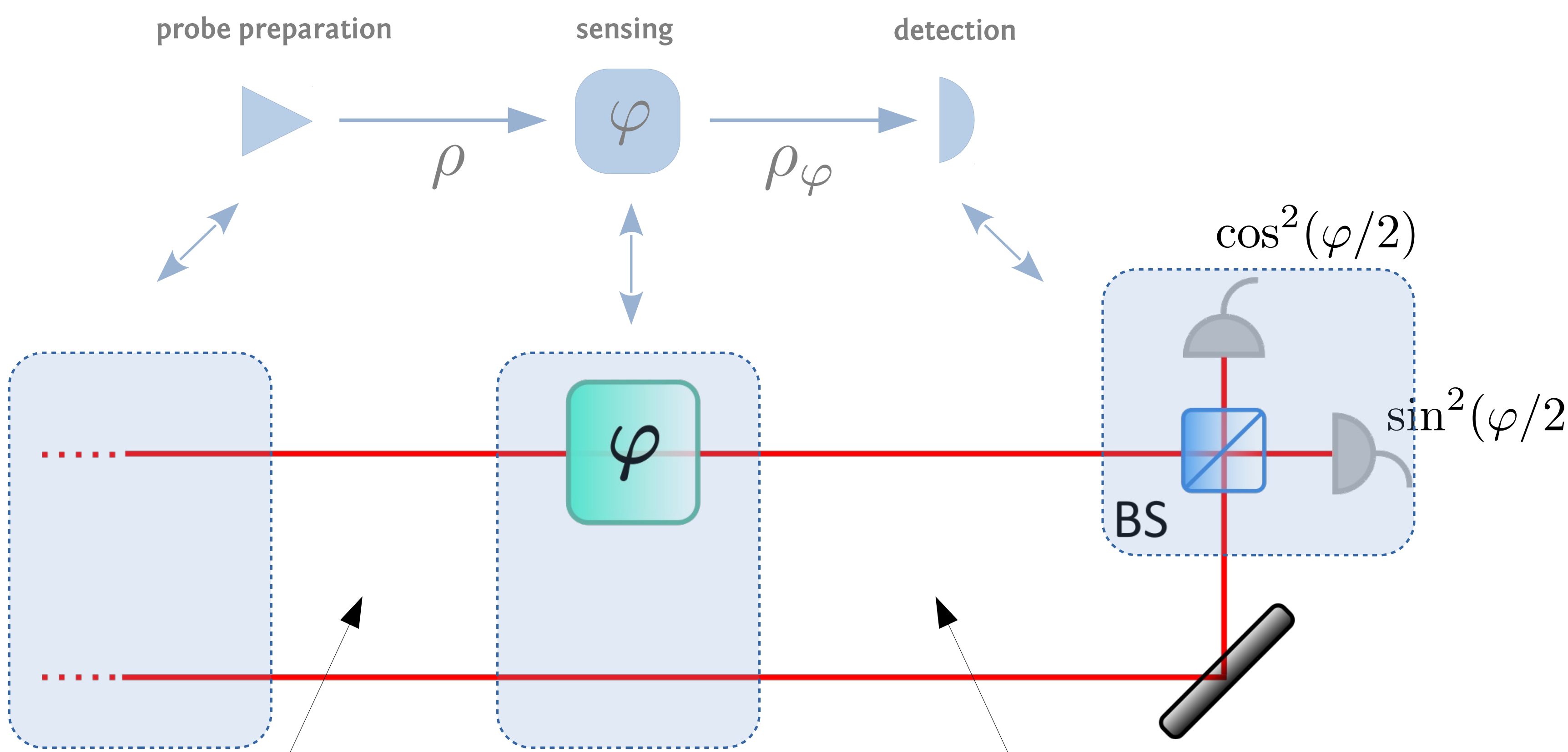

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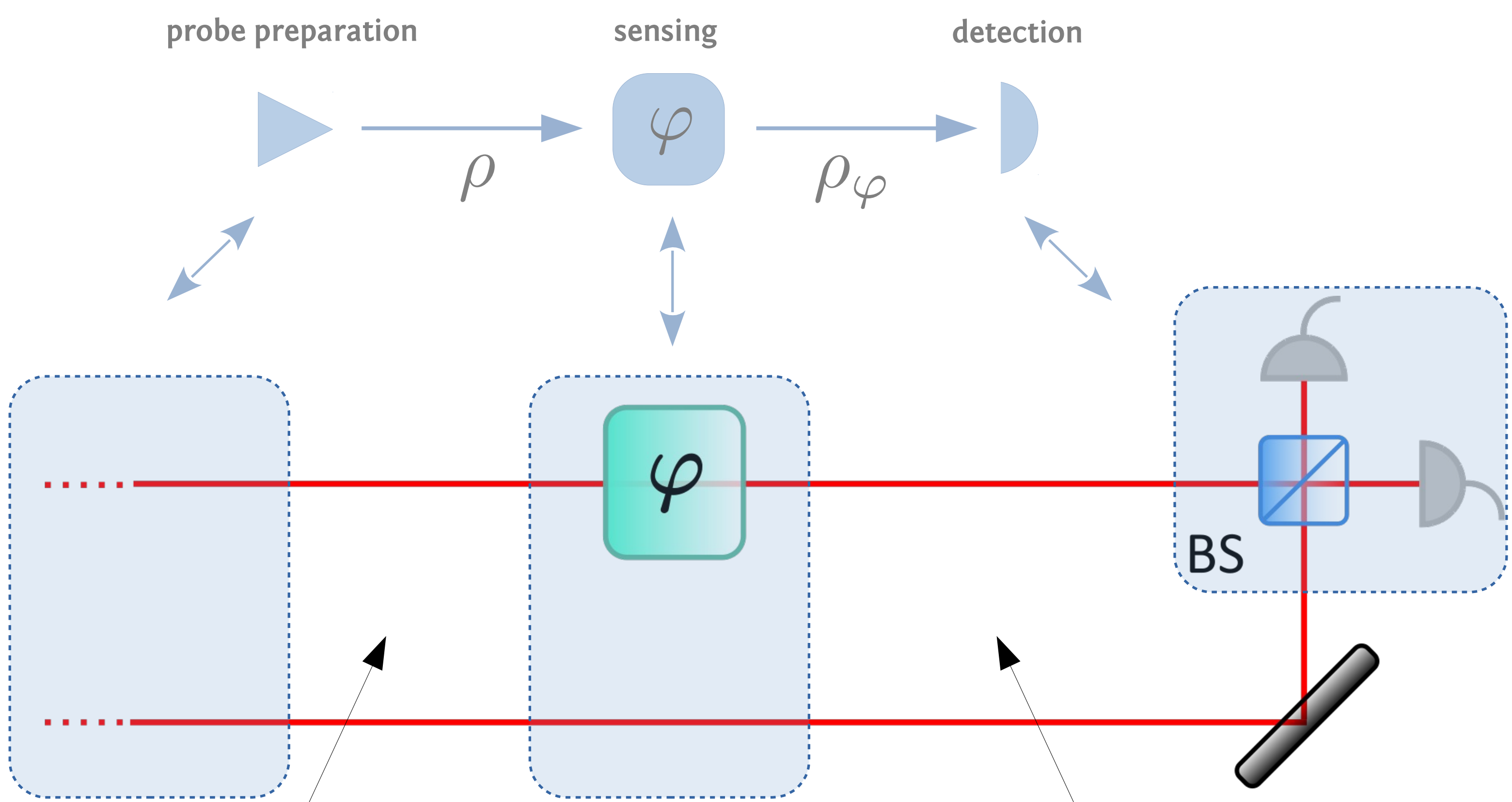

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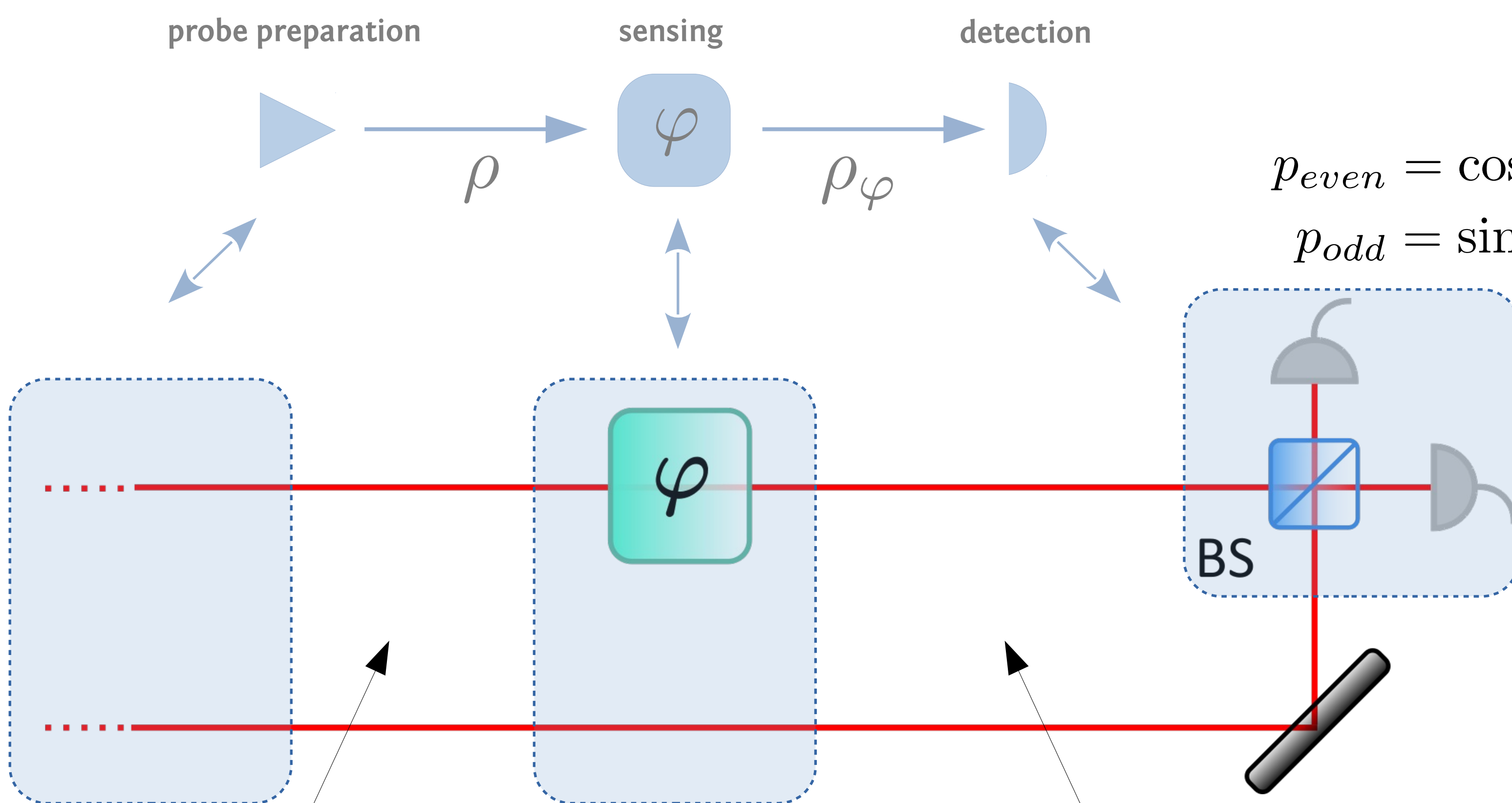

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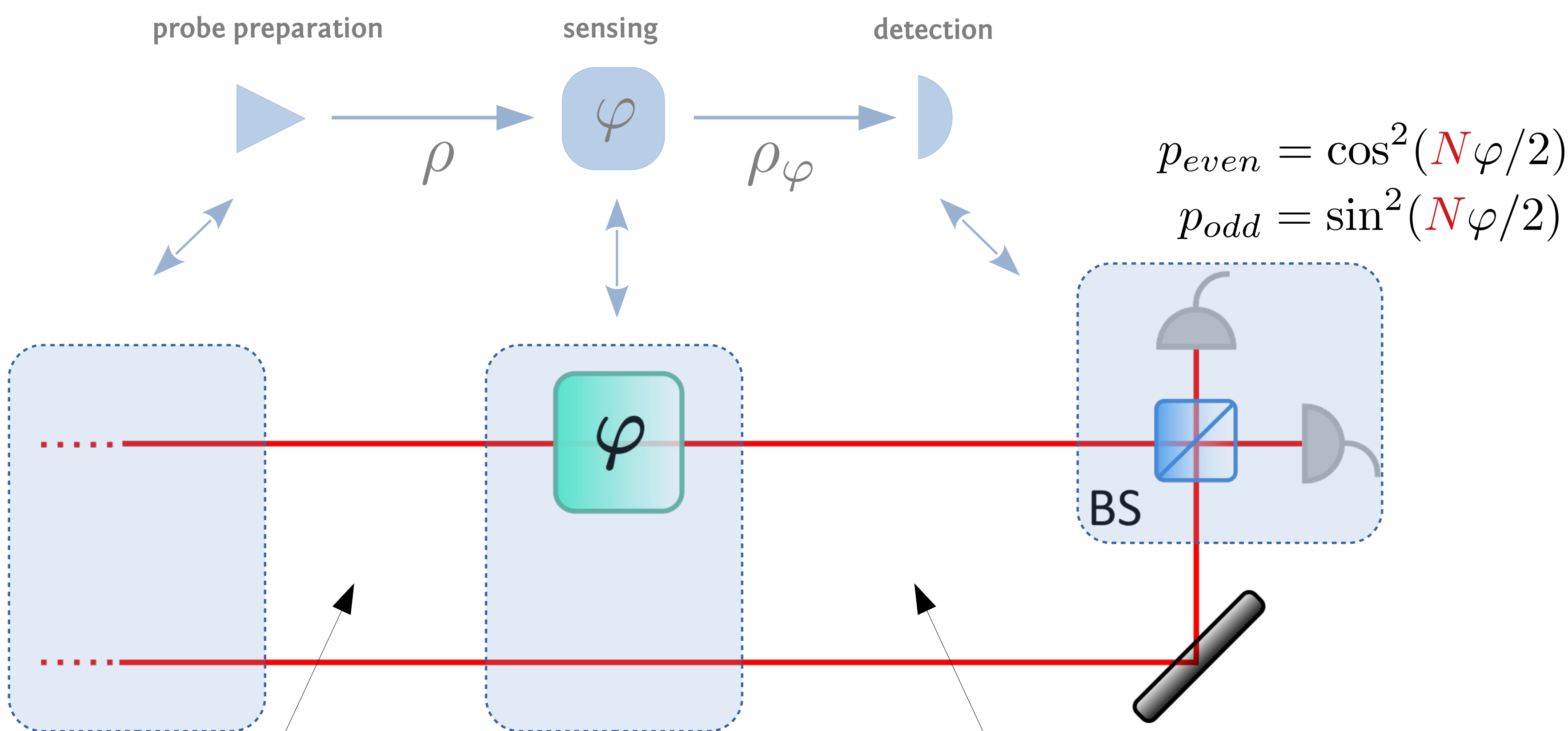

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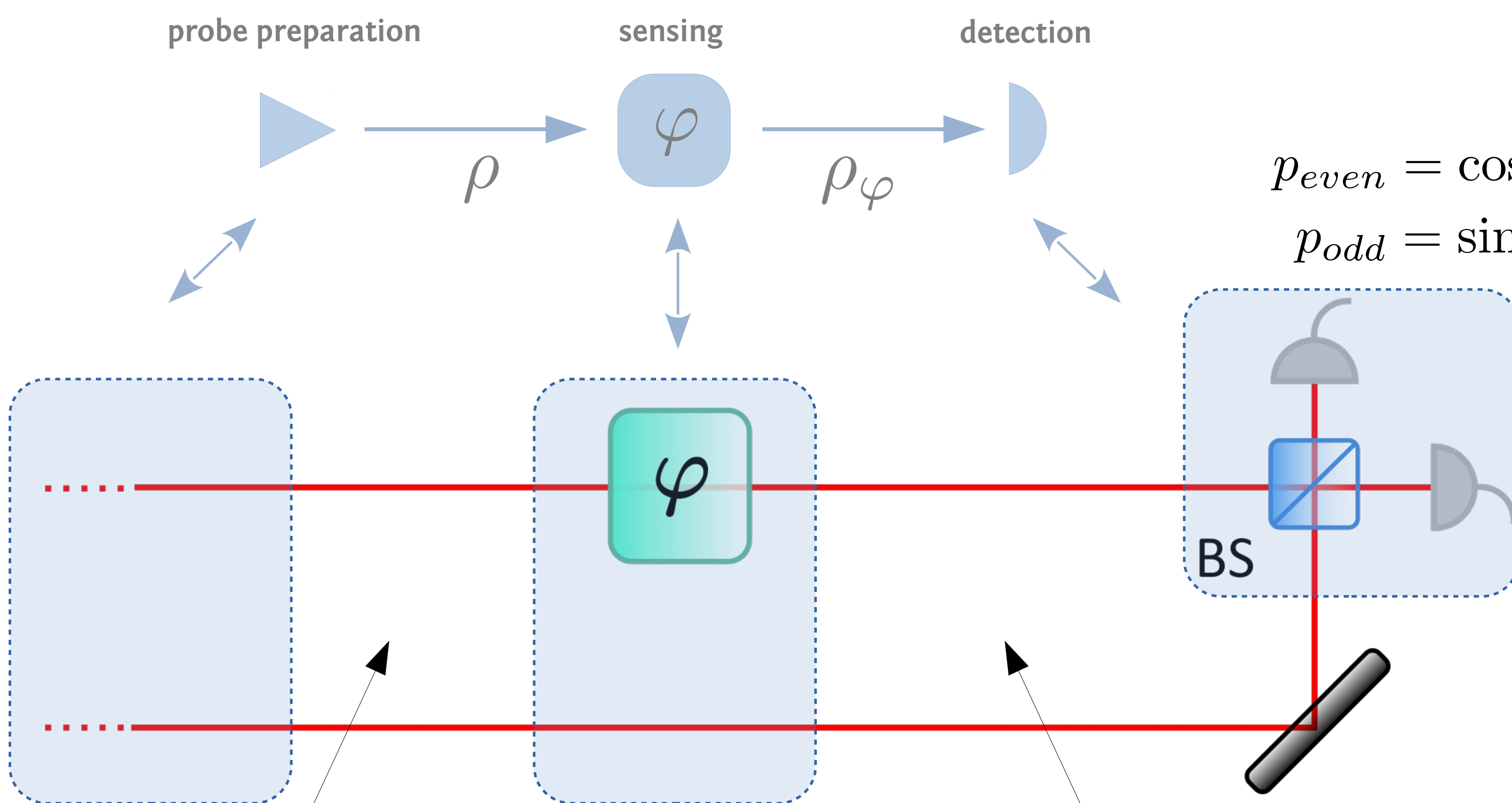

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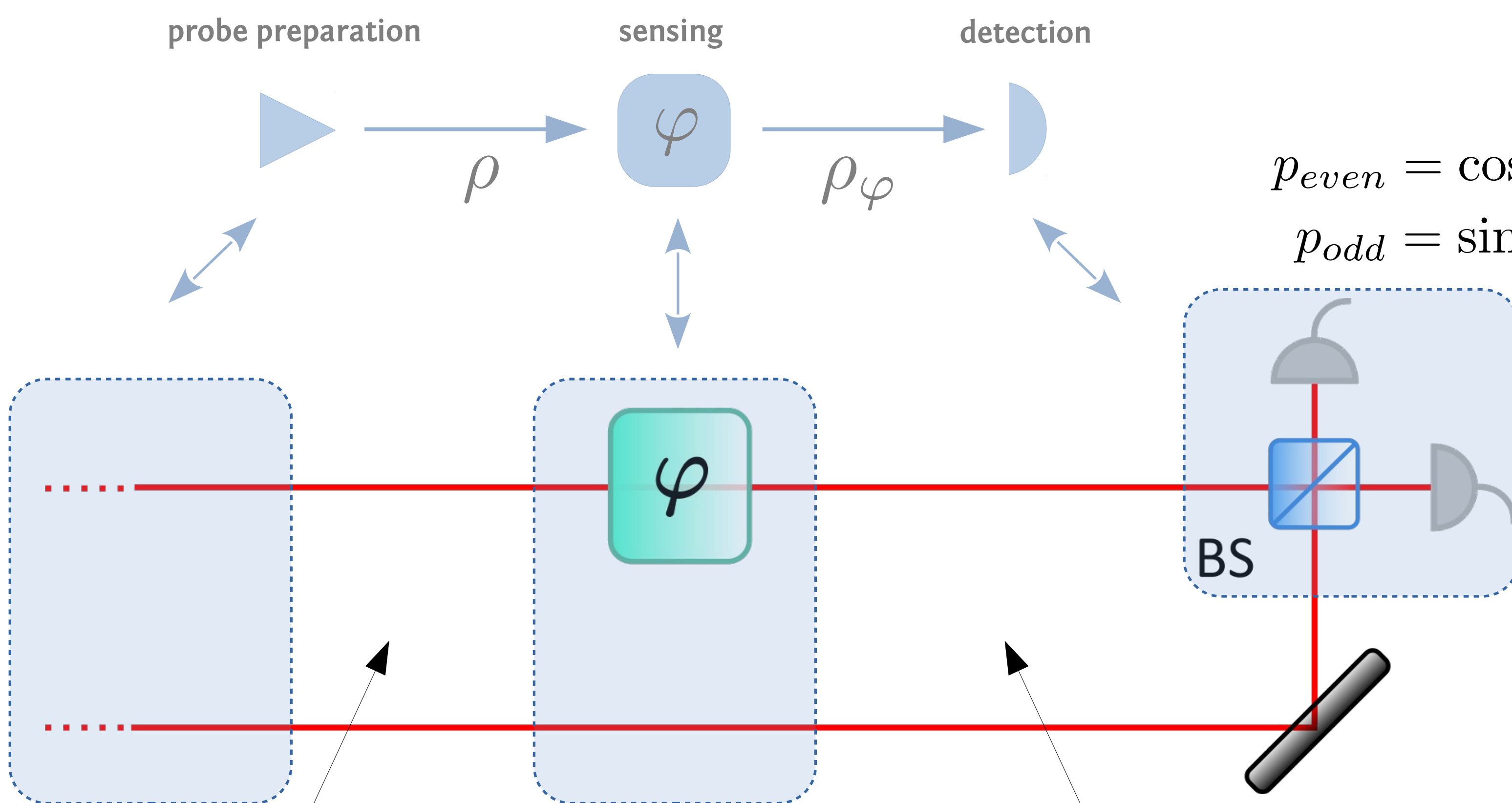
Achieving the Heisenberg limit – optical phase estimation



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Quantum strategy

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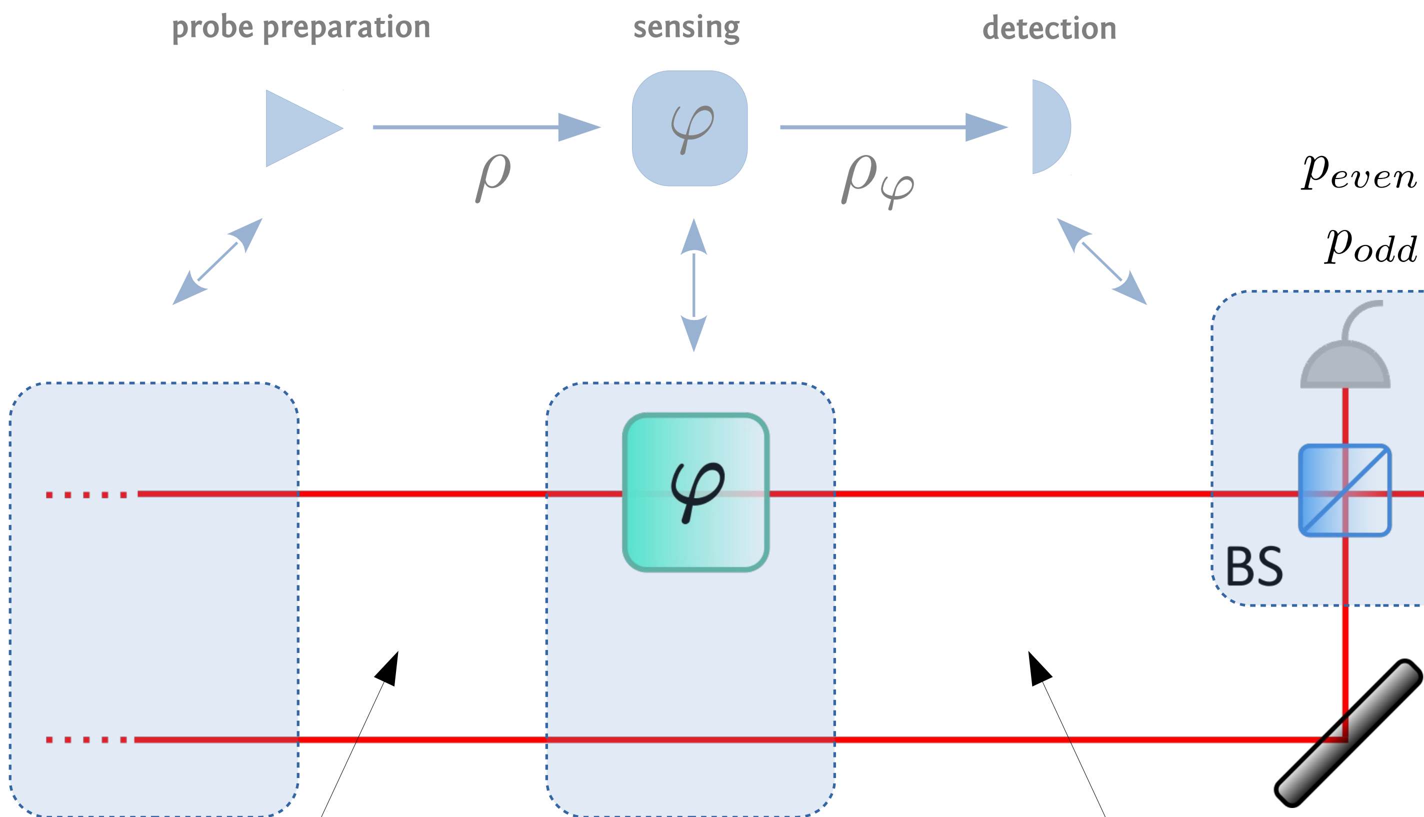
Achieving the Heisenberg limit – optical phase estimation



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$$\Rightarrow \quad \Delta^2\varphi = \frac{1}{\nu N}$$

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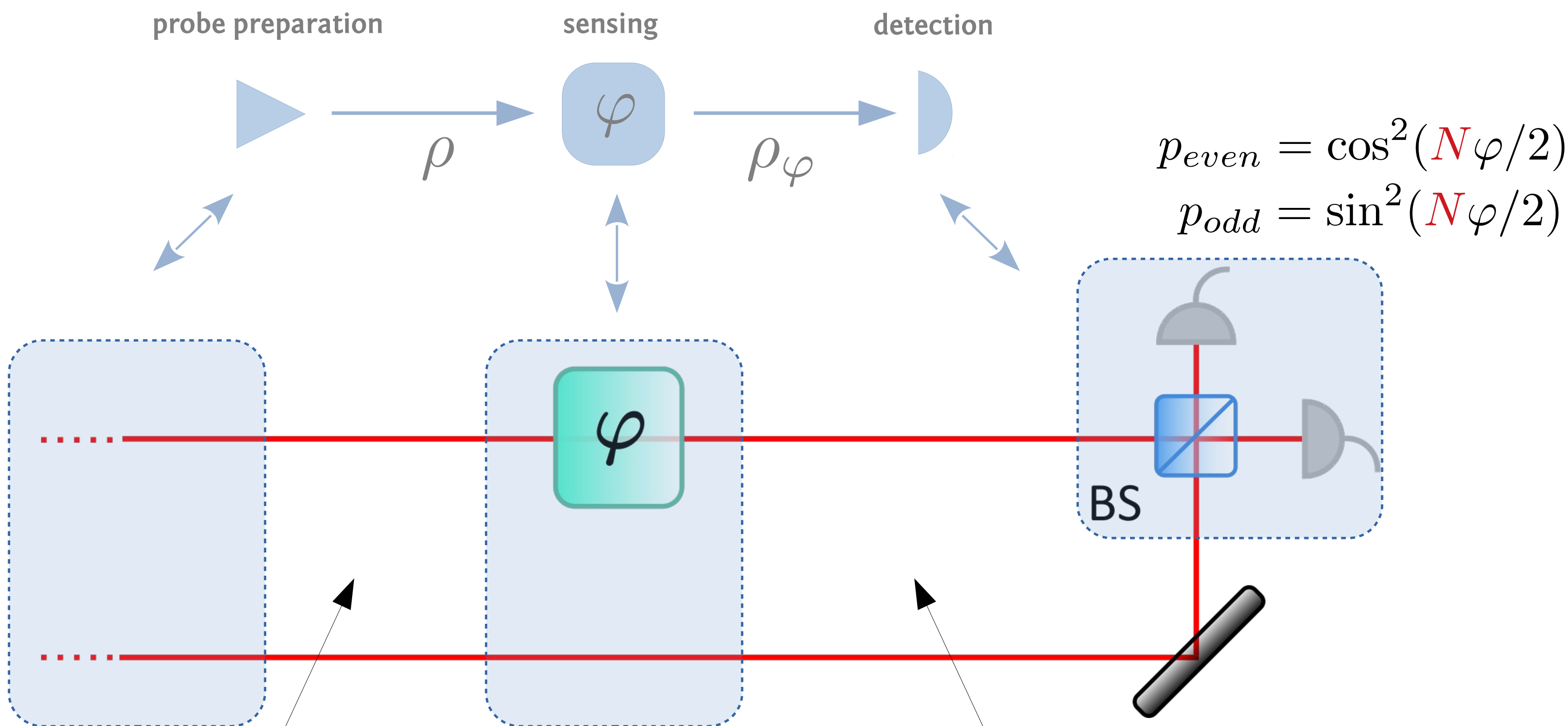

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Quantum strategy

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repeat ν times

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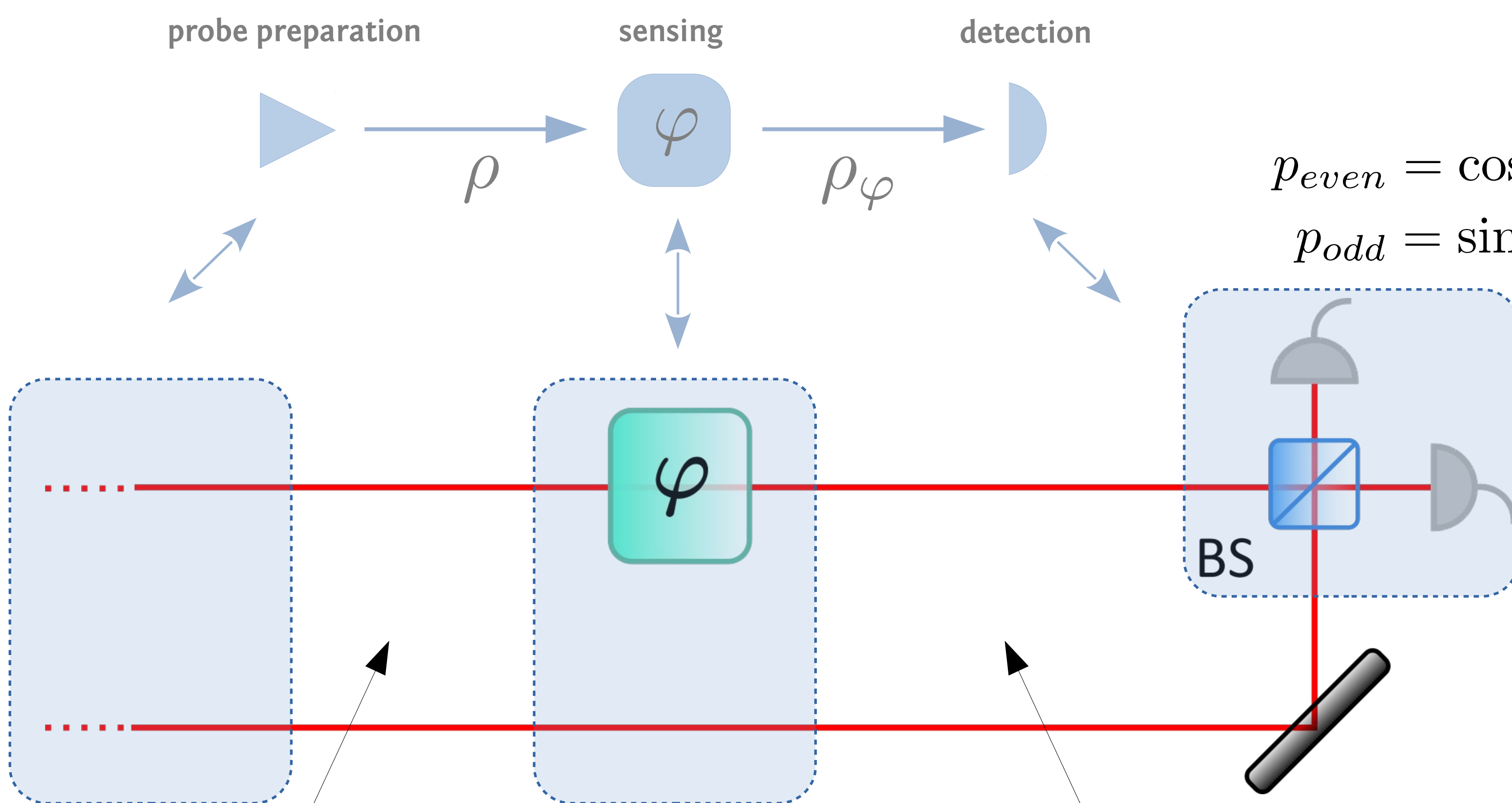
Achieving the Heisenberg limit – optical phase estimation



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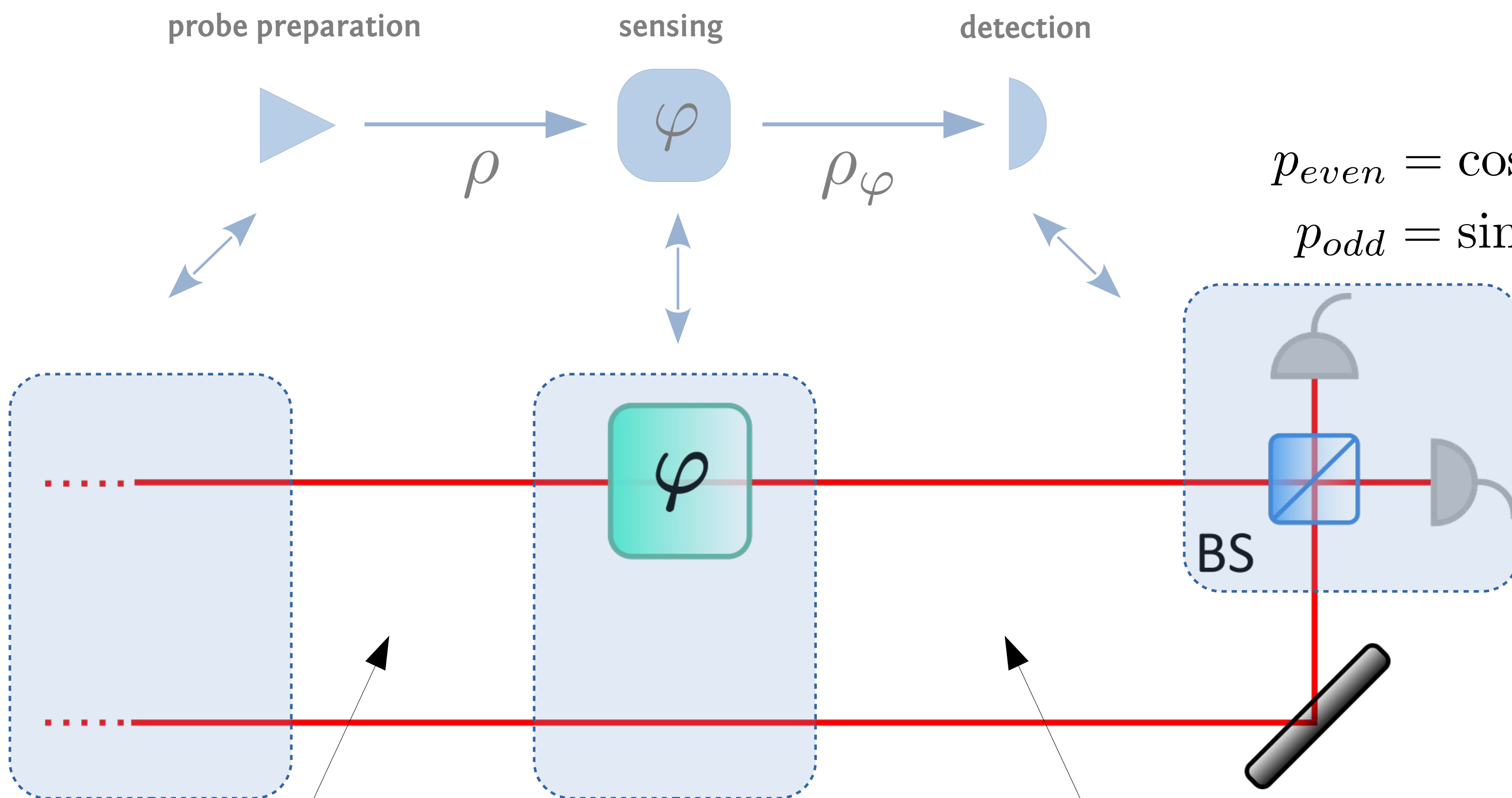

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Quantum strategy

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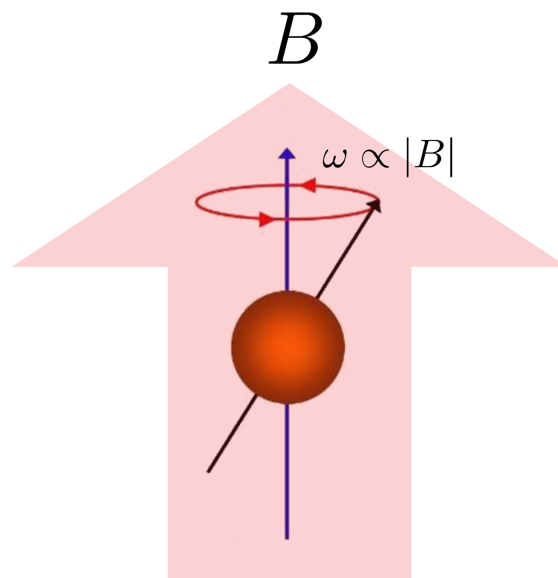
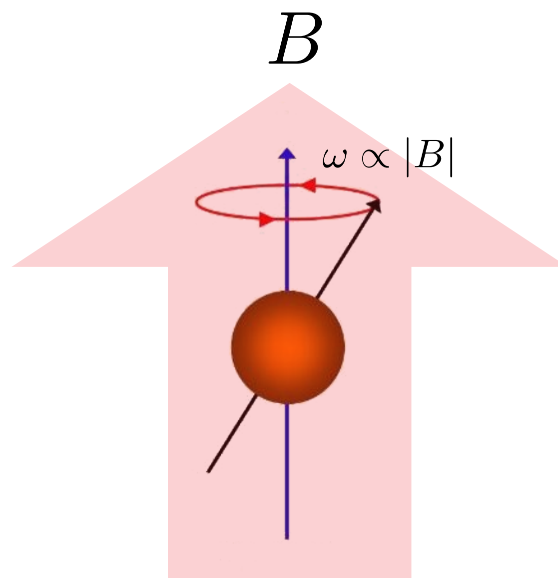
Achieving the Heisenberg limit - magnetometry



$$H = \frac{1}{2}\omega\hat{\sigma}_z$$

Achieving the Heisenberg limit - magnetometry

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$$



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Achieving the Heisenberg limit - magnetometry

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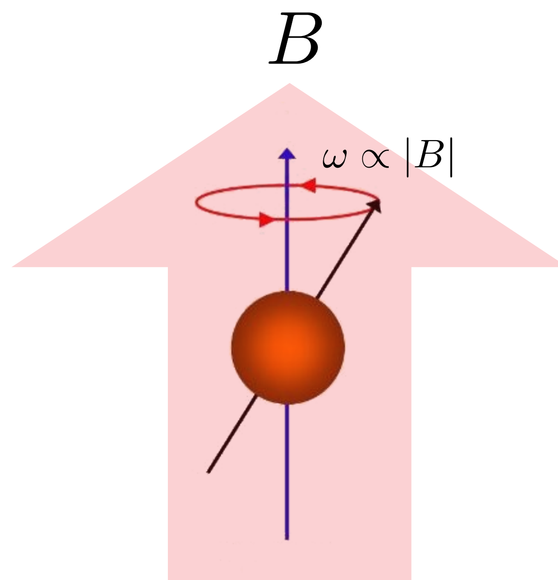

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Achieving the Heisenberg limit - magnetometry

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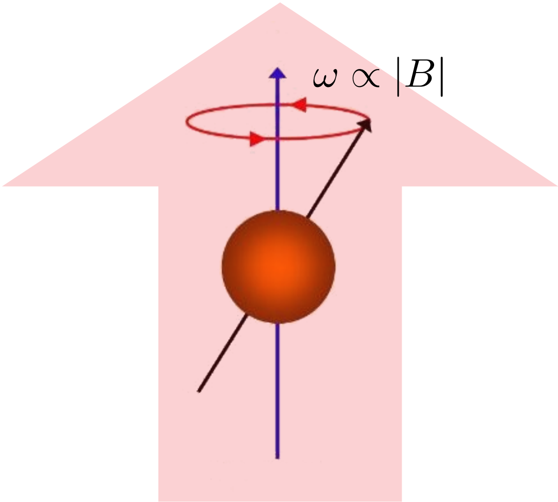
$\hat{\sigma}_x$

$$P_+ = \cos^2(\omega\tau/2)$$

B



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Achieving the Heisenberg limit - magnetometry

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$$

$$\downarrow$$

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$$\downarrow \hat{\sigma}_x$$

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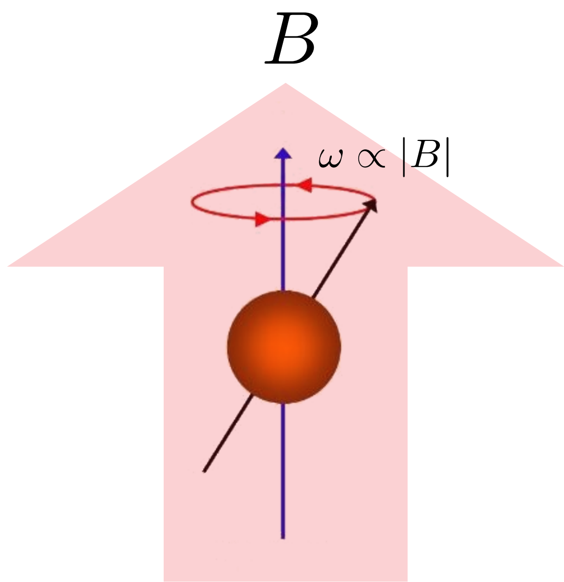

$$H = \frac{1}{2}\omega\hat{\sigma}_z$$

fix total measurement time T

use N particles $N(T/\tau)$ times

Achieving the Heisenberg limit - magnetometry

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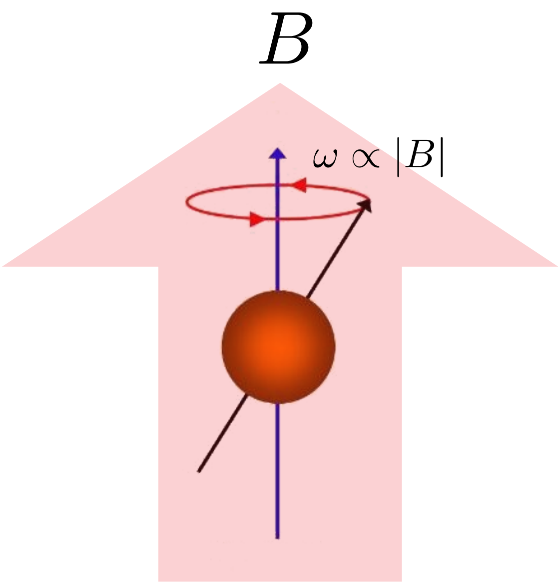

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Classical strategy

$$\mathcal{F}_\omega = \tau^2$$

$$\Delta^2\omega = \frac{1}{N\tau^2(T/\tau)} = \frac{1}{N\tau T}$$

Achieving the Heisenberg limit - magnetometry

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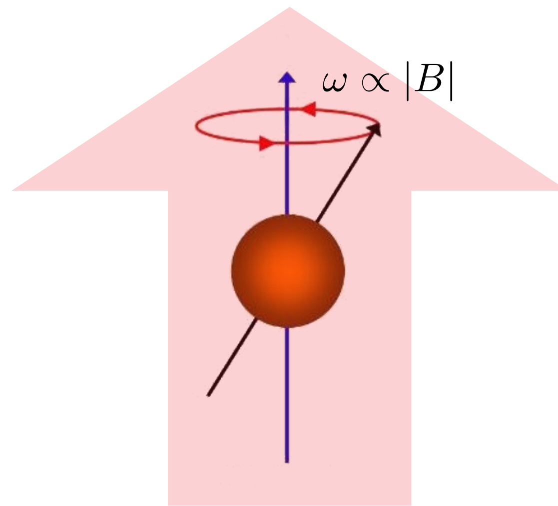

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Optimal to maximise interaction time

Achieving the Heisenberg limit - magnetometry

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$$

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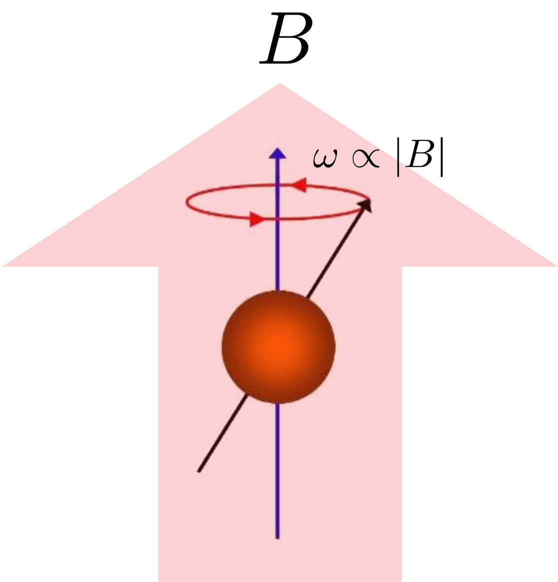

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fix total measurement time T

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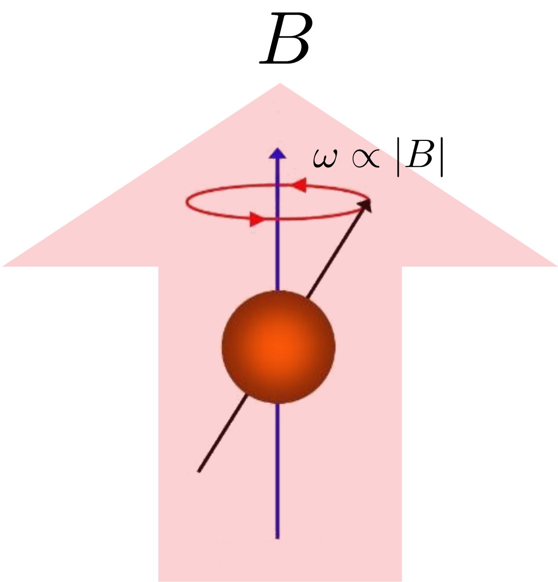

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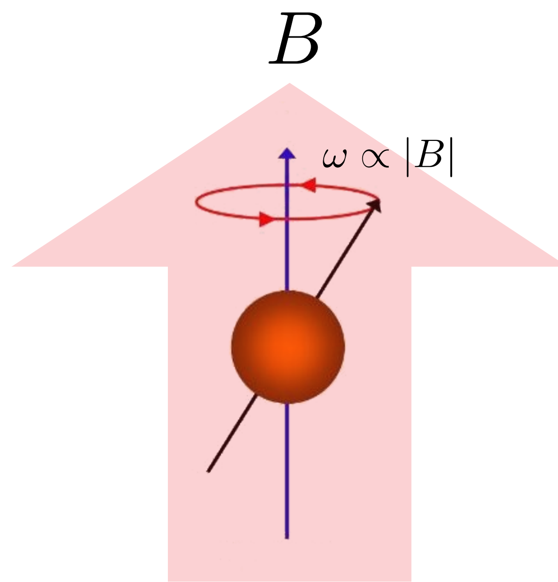

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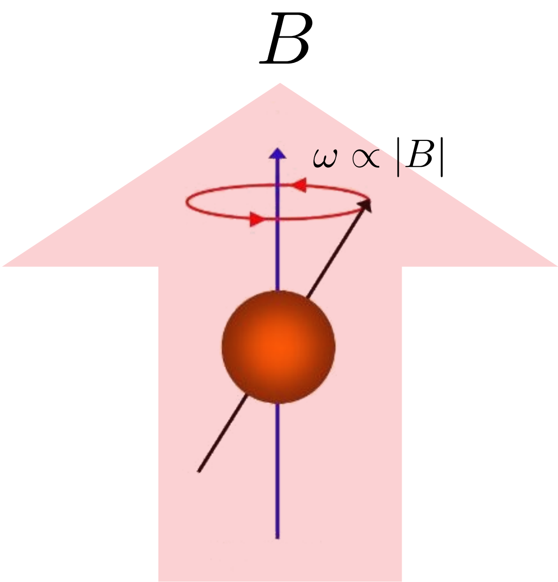

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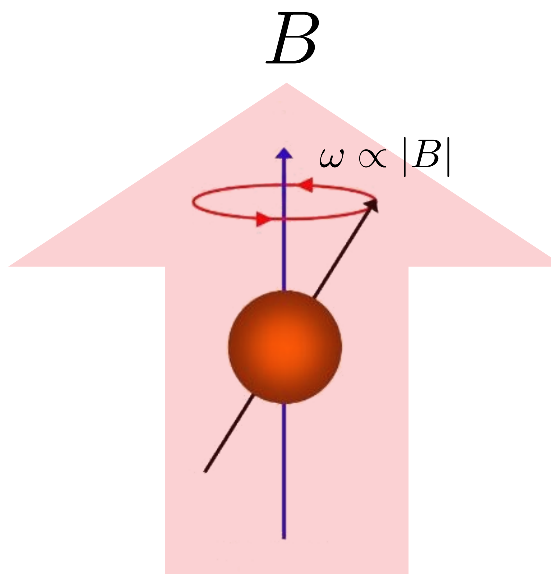

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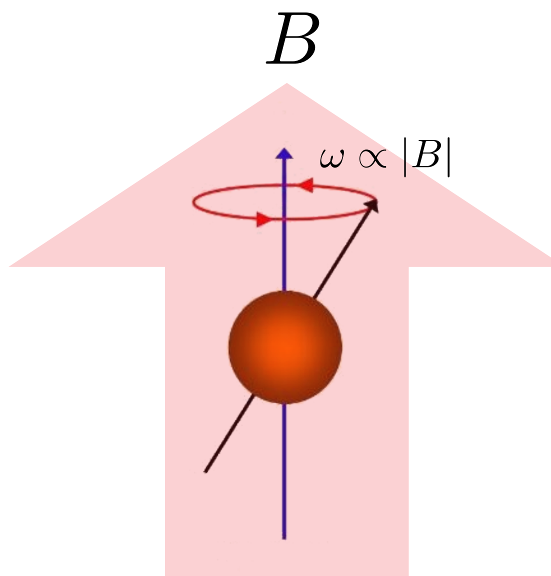

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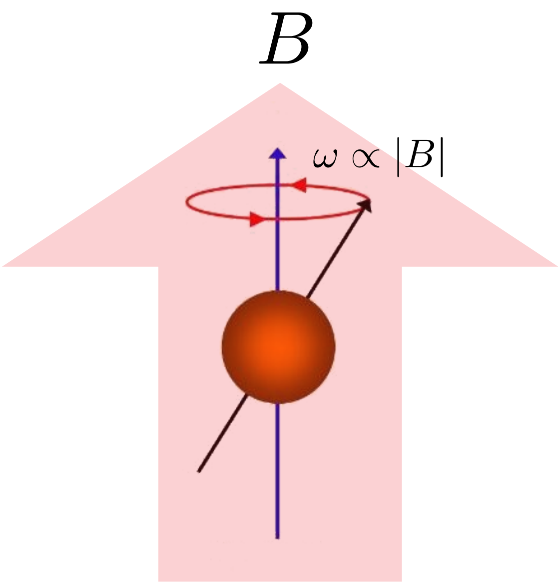

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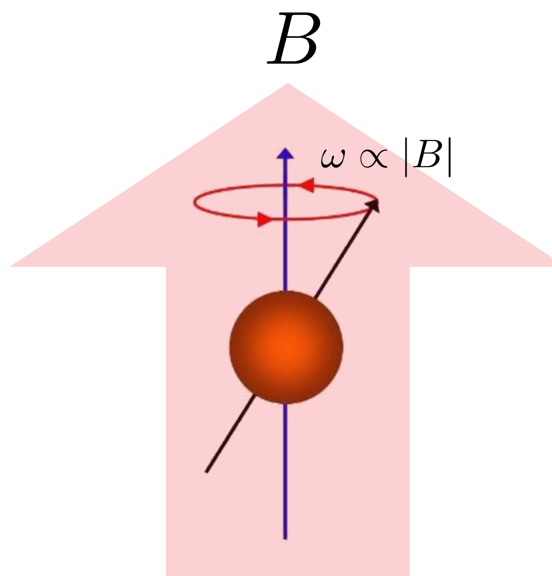

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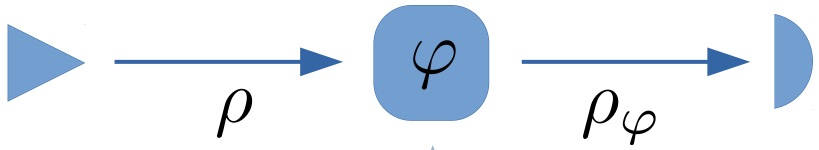
Achieving the Heisenberg Limit - general noiseless case



Unitary encoding of parameter.

$$U_\varphi = e^{-i\varphi H}$$

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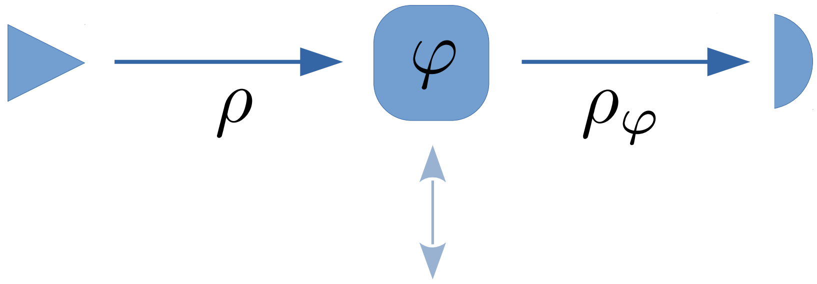

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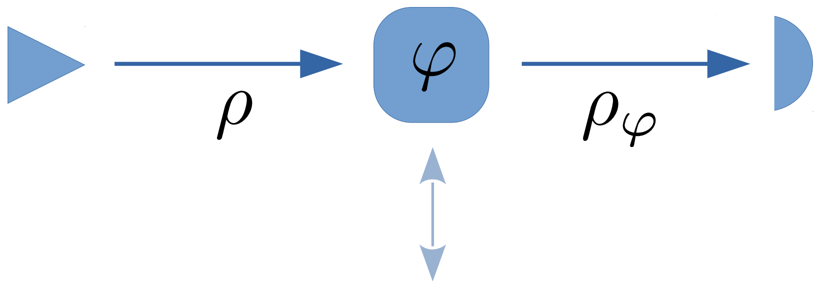

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QFI convex → maximised on pure states.

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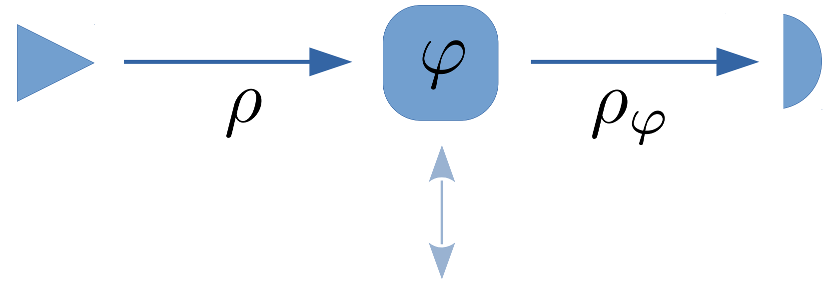

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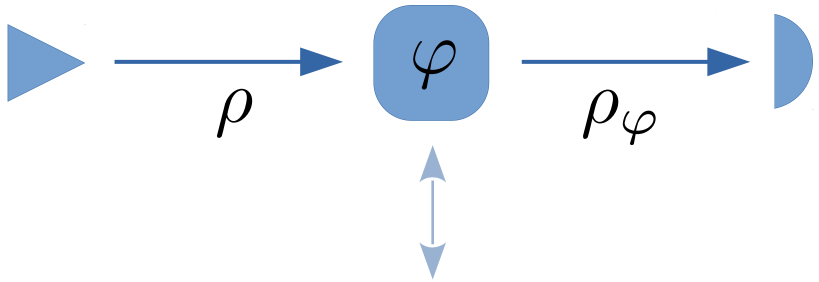

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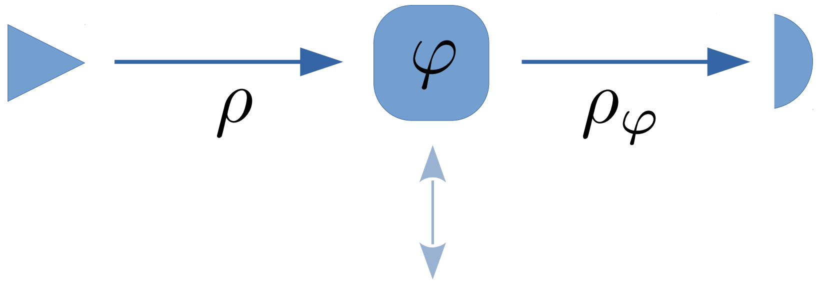

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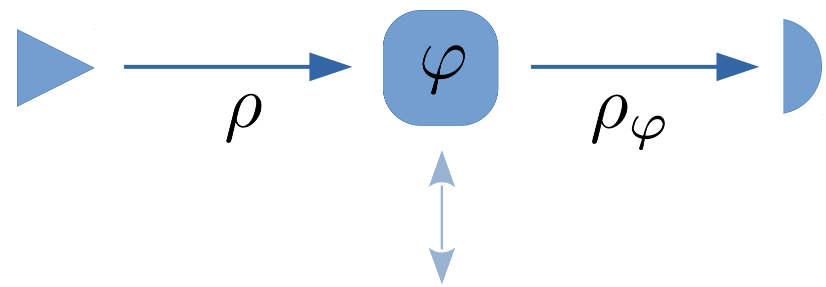

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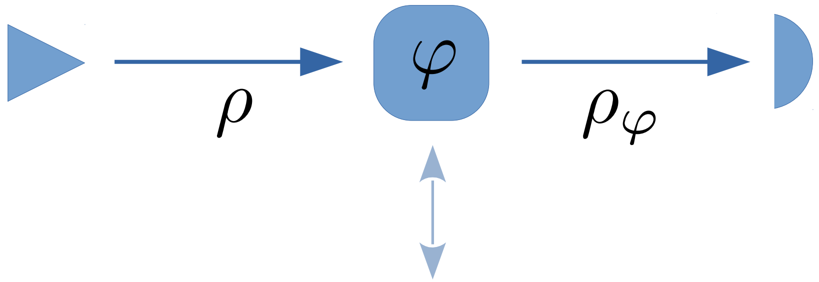

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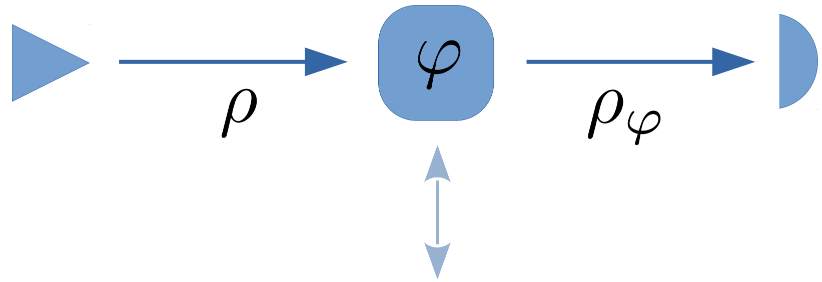

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$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\lambda_{min}\rangle + |\lambda_{max}\rangle) \quad \Rightarrow \quad \Delta^2\varphi = \frac{1}{\nu N^2(\lambda_{max} - \lambda_{min})}$$

10 MIN BREAK

Precision scaling with noise

Noise might be…

- Fluctuating background fields.
- Interferometer instability.
- Laser noise (power, phase,…)
- Imperfect state preparation.
- Imperfect detectors.
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Unitary evolution → Quantum channel

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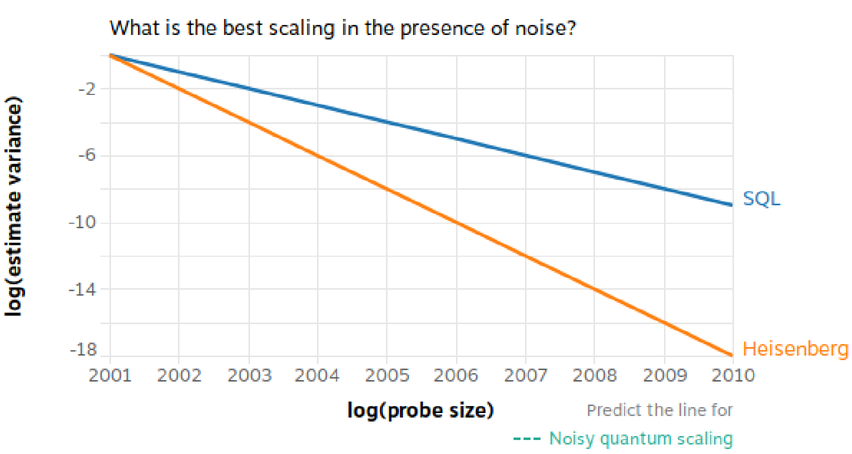
What do you think the best scaling with noise is?

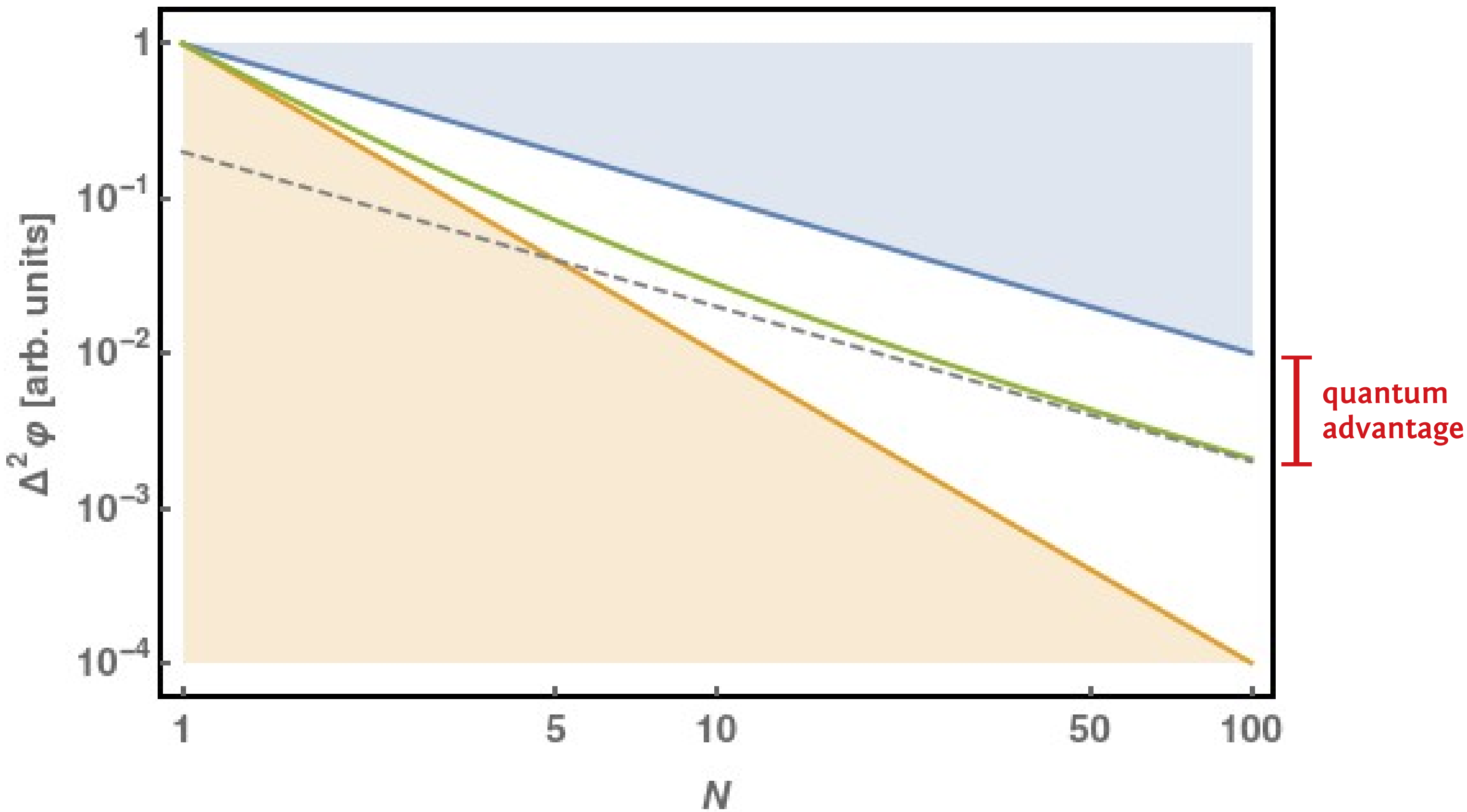
jonatanbohrbrask.dk/scaling2



No-go result for scaling advantage

At most a constant factor for any full-rank channel

'full rank' ~ no subspace is completely free of decoherence



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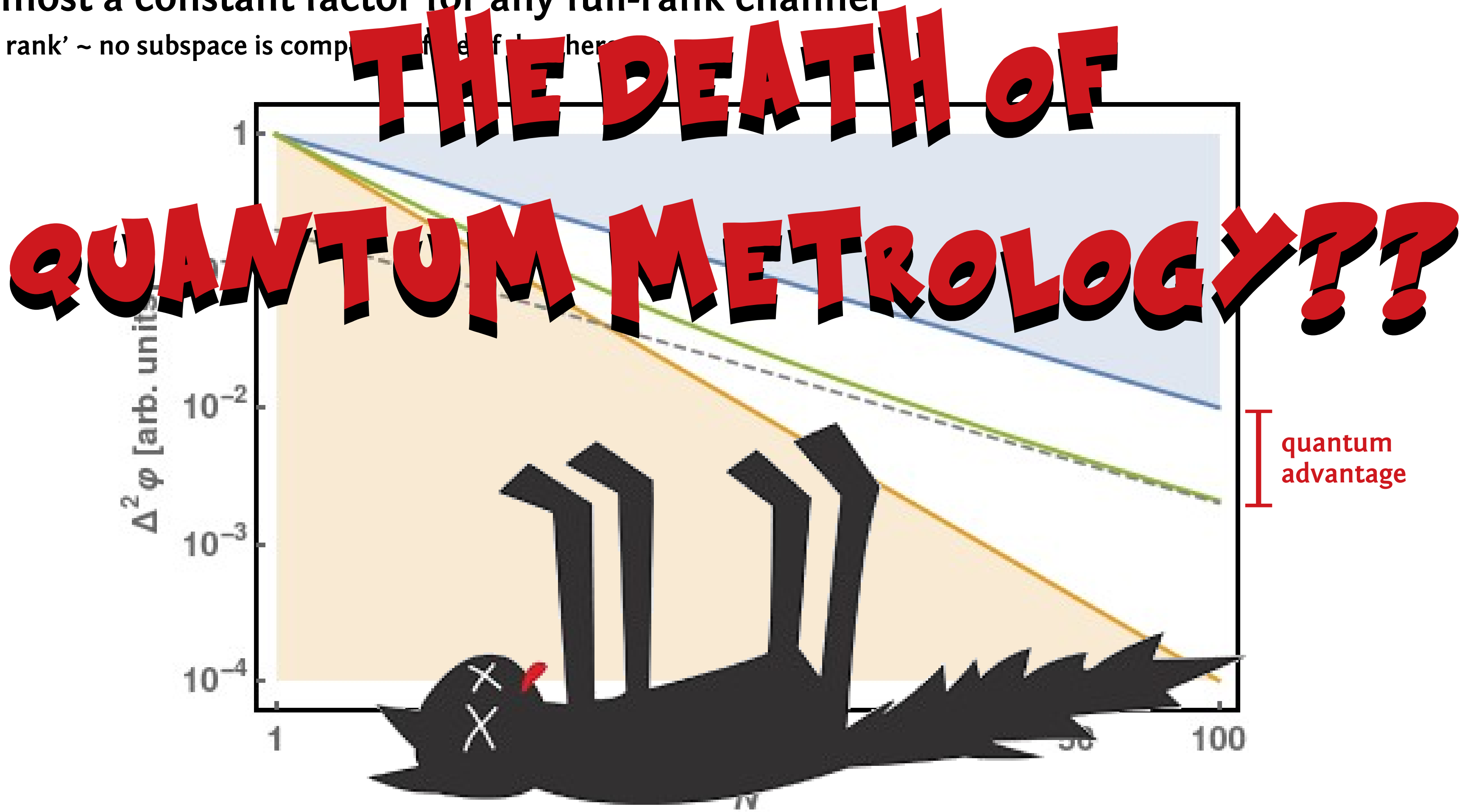

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Scaling advantages are possible for directional noise.

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The Heisenberg limit is attainable for certain noise using error correcting codes

The finite-size regime is important

Improvements when classical precision has been pushed to the limit

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LIGO: Cannot increase light power much further.


NEWS QUANTUM PHYSICS, ASTRONOMY, GRAVITATIONAL WAVES

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Quantum 'squeezing' light could lead to daily gravitational wave detections

BY EMILY CONOVER 2:54PM, FEBRUARY 15, 2019

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LIGO LAB/MIT/CALTECH

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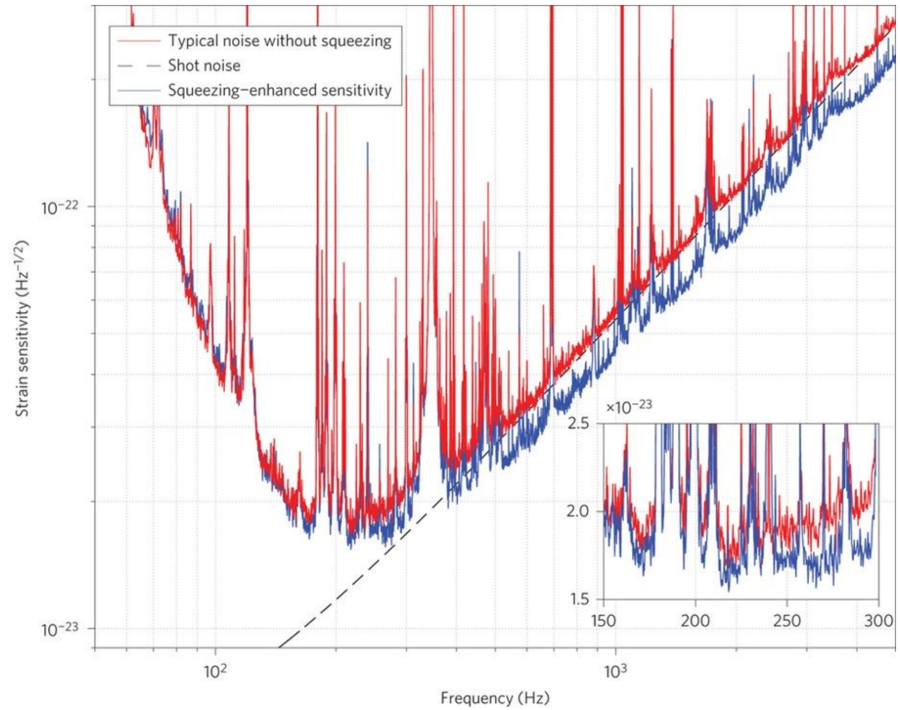

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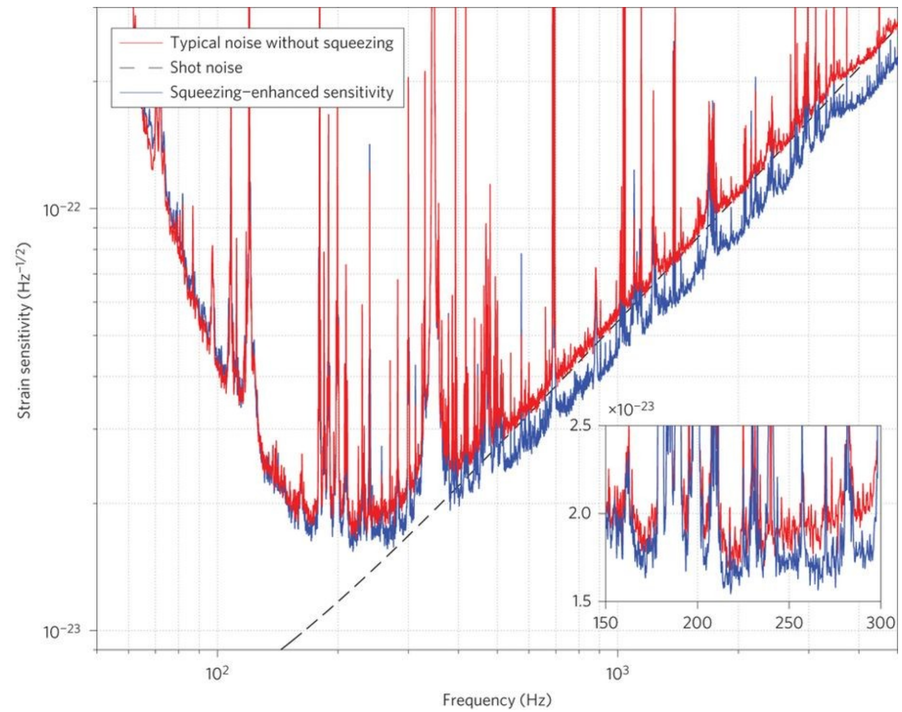

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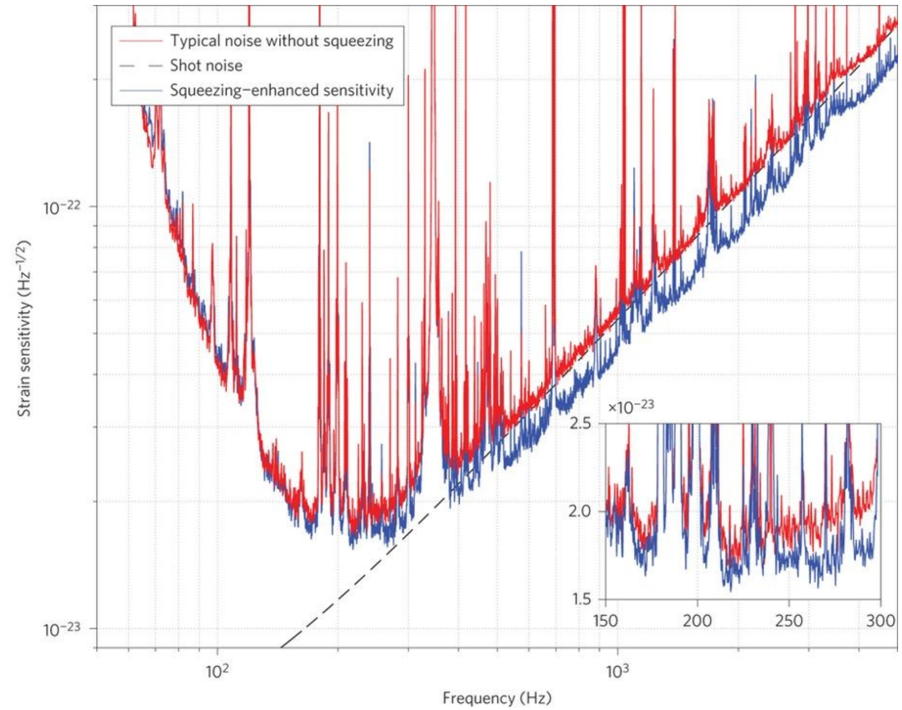

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Can enable smaller / faster sensors – better precision without increasing probe size

Imporant e.g. in biological applications.

Quantum scaling advantage for directional noise

Markovian noise described by master equation

$$\dot{\rho} = -i\omega[H, \rho] - \sum_m \gamma_m \left(L_m \rho L_m^\dagger - \frac{1}{2}\{L_m^\dagger L_m, \rho\} \right)$$

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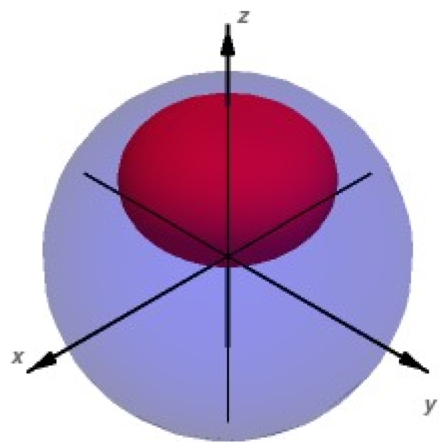

general

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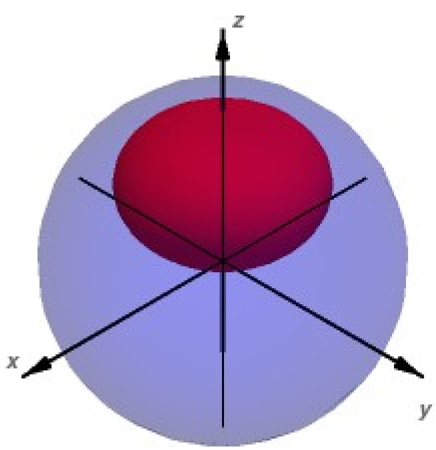
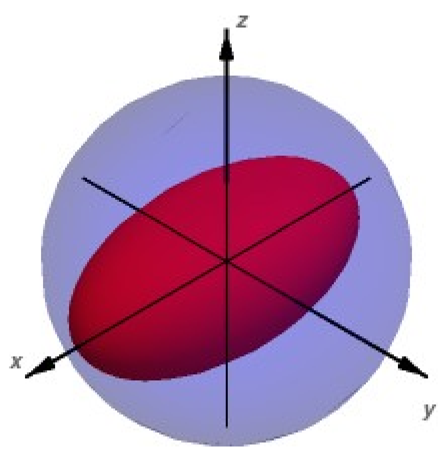


general



directional

$$\gamma_x \neq 0$$
$$\gamma_z, \gamma_y = 0$$

Optimise interaction time for each *N*

Frequency estimation under perpendicular noise

$$\tau_{opt} \sim N^{-1/3} \quad \Rightarrow \quad \Delta^2\omega \sim \frac{1}{N^{5/3}}$$

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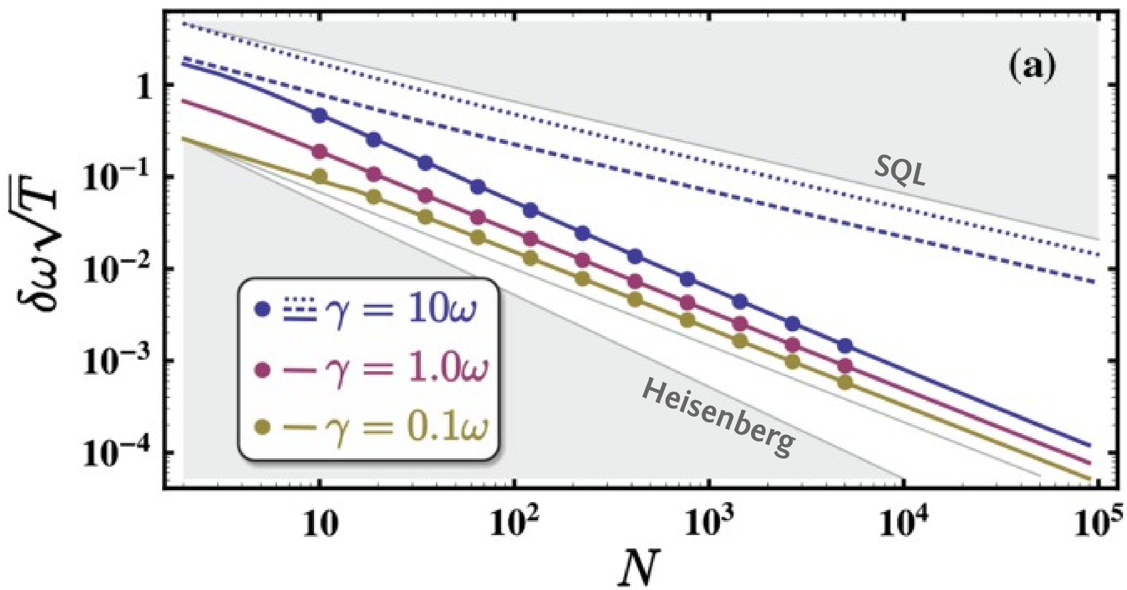

Chaves, JBB *et al.*, *Phys. Rev. Lett.* 111, 120401 (2013)

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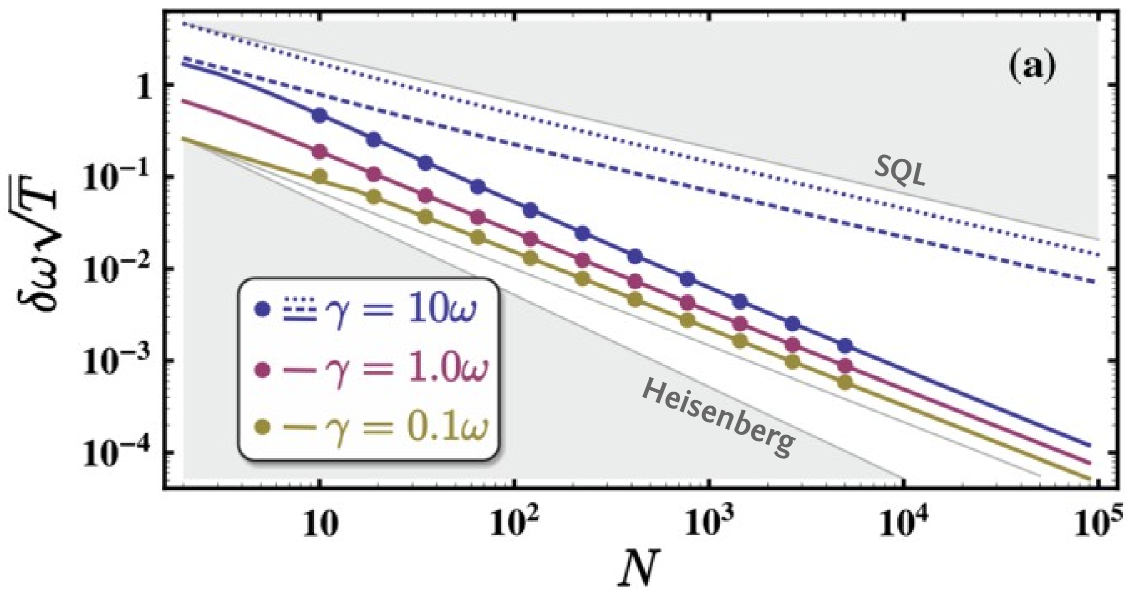
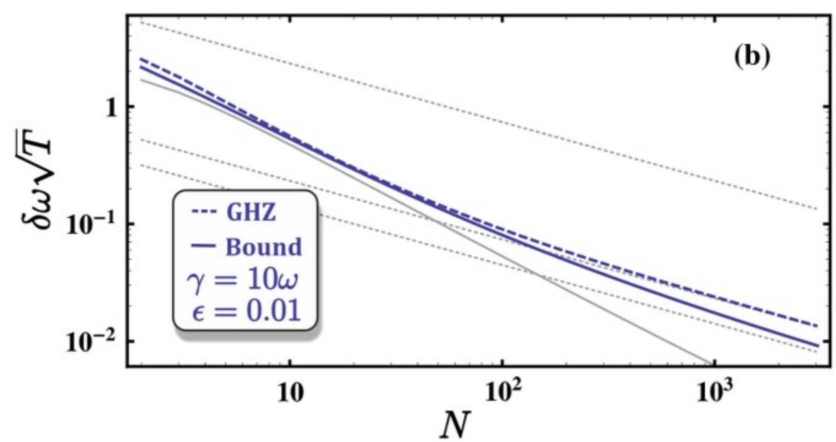


noise not perfectly parallel / interaction time lower bounded
→ constant factor asymptotic scaling

...but cross over can be at large *N*.



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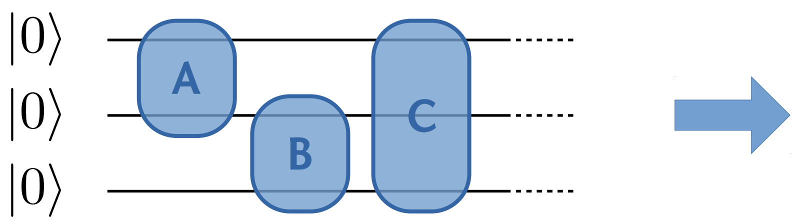
Heisenberg scaling with error correction

Error correcting codes

Encode logical qubits in multiple noisy physical qubits



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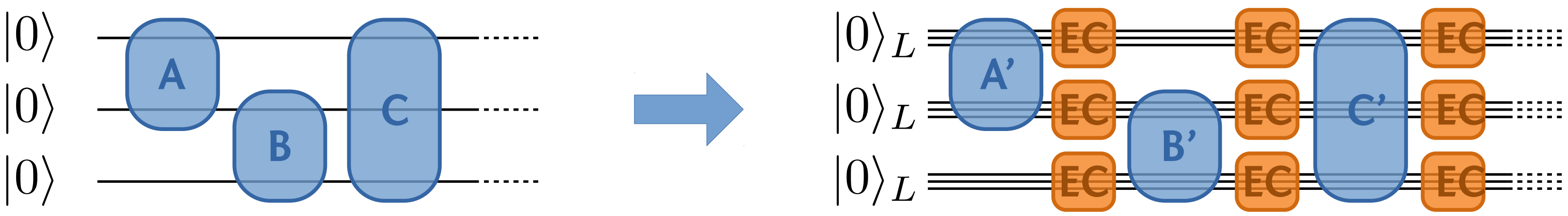

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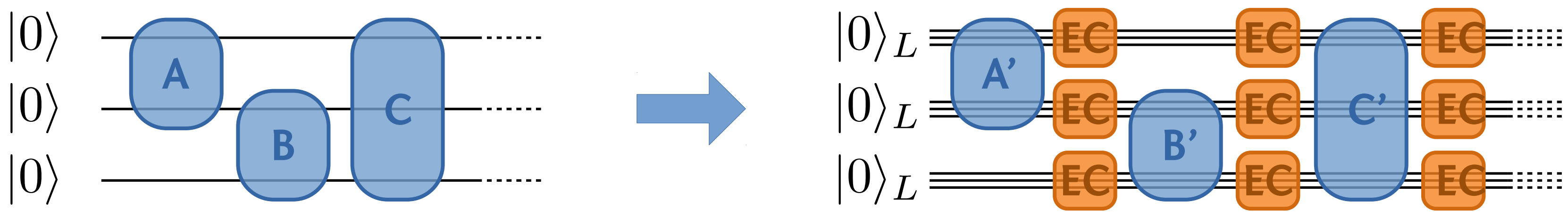

E.g. correct bit flips with majority voting

$$|0\rangle_L = |0\rangle|0\rangle|0\rangle$$

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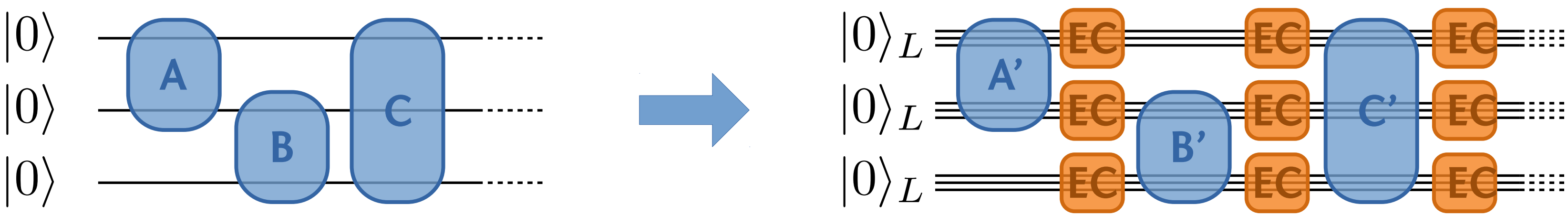

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$$|0\rangle_L = |0\rangle|0\rangle|0\rangle \quad \xrightarrow{\hat{\sigma}_x^{(2)}} \quad |0\rangle|1\rangle|0\rangle$$

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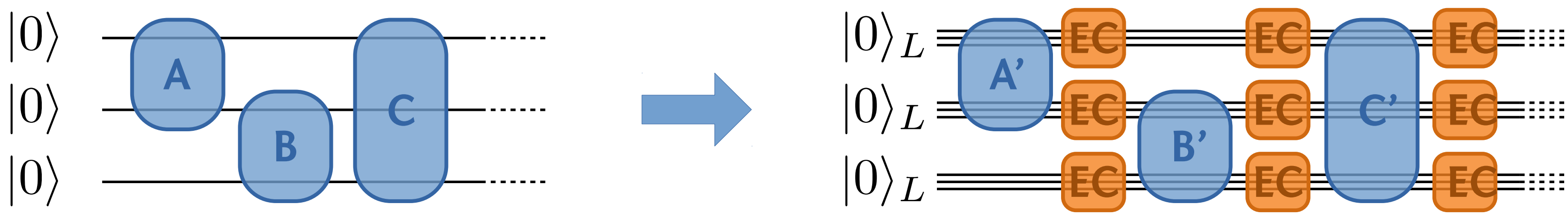

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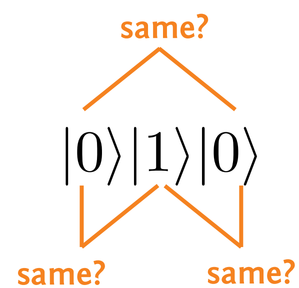

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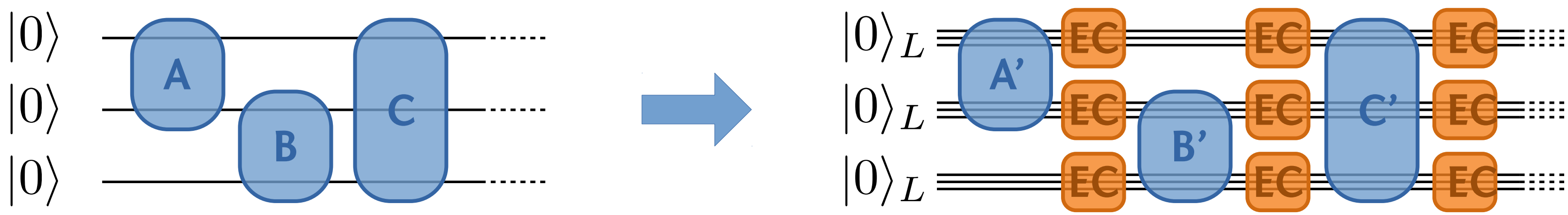

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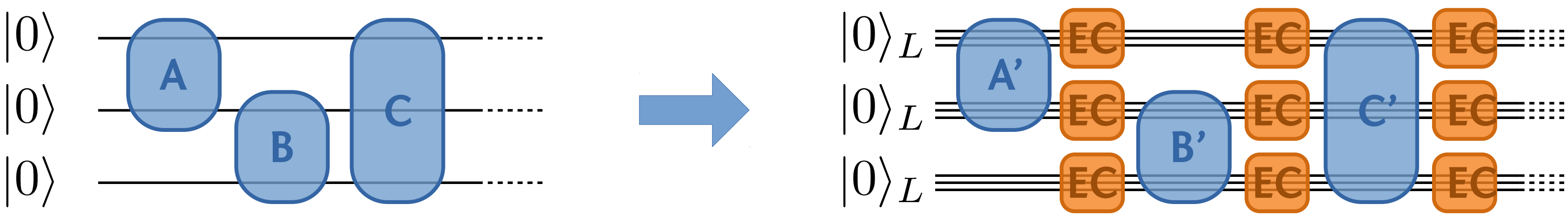

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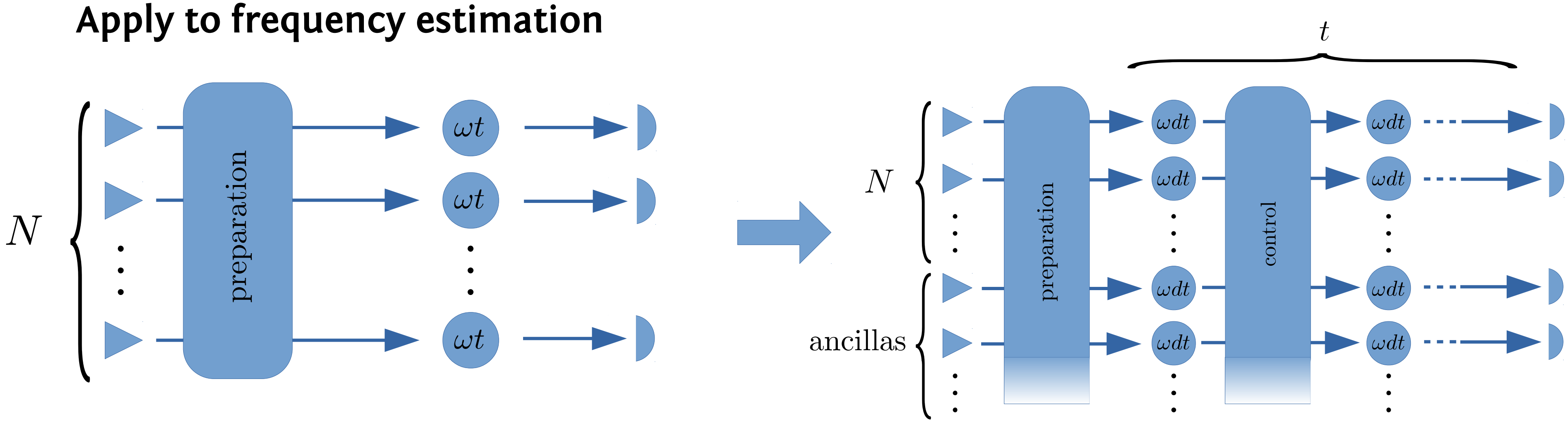
Apply to frequency estimation



Intuitively: can correct errors if the correction does not also 'correct' the signal

Zhou *et al.*, *Nature Communications 9, 78 (2018)*

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$$\dot{\rho} = -i\omega[H, \rho] - \sum_m \gamma_m \left(L_m \rho L_m^\dagger - \frac{1}{2}\{L_m^\dagger L_m, \rho\} \right)$$

Heisenberg scaling is possible *if and only if* Hamiltonian not in Lindblad span

$$H \notin \mathrm{span}\{L_k\}$$

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For qubits: a single Lindblad operator not parallel to Hamiltonian

$$H = \frac{1}{2}\hat{\sigma}_z \qquad L_x = \hat{\sigma}_x$$

Zhou *et al.*, *Nature Communications 9, 78 (2018)*

Some literature

Quantum metrology

- https://science.sciencemag.org/content/306/5700/1330.abstract
- https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.96.010401
- https://www.nature.com/articles/nphoton.2011.35
- https://arxiv.org/abs/1807.11882

No-go results

- https://iopscience.iop.org/article/10.1088/1751-8113/41/25/255304
- https://www.nature.com/articles/nphys1958
- https://www.nature.com/articles/ncomms2067
- https://iopscience.iop.org/article/10.1088/1367-2630/15/7/073043

Directional noise

- https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.111.120401
- https://journals.aps.org/prx/abstract/10.1103/PhysRevX.5.031010

Error correction

- https://www.nature.com/articles/s41467-017-02510-3
- https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.112.150802
- https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.112.150801
- https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.112.080801
- https://quantum-journal.org/papers/q-2017-09-06-27/